### The Intrinsic Dimensionality of Images and its Impact on Learning

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# Learning problem that require exponential sample complexity in manifold dimension

Learning general boolean functions on the d-hypercube (need ALL 2<sup>d</sup> vertices)

Learning a smooth class boundary on a manifold to ε-accuracy (Narayanan & Niyogi, COLT 2009)

Learning a manifold with bounded extrinsic curvature to ε-accuracy (Narayanan & Mitter, NeurIPS 2010)





## AND...YET

#### We "solved" ImageNet Learning a smooth function in high dimensions





#### GANs Learning a "manifold"

StyleGAN, Karras 2018

## QUESTIONS

Can we explicitly measure the lowdimensional structure of image manifolds?

Just how low-dimensional are they?

What is the impact of dimension on learning?

## DIMENSIONALITY ESTIMATION

Maximum Likelihood Estimation (Levina & Bickel, NeurIPS '04)

 $T_k(x)$  = "Distance from point x to k<sup>th</sup> NN"

Local Estimate:

$$\hat{m}_k(x) = \left[\frac{1}{k-1} \sum_{j=1}^{k-1} \log \frac{T_k(x)}{T_j(x)}\right]^{-1}$$



We compute the average estimator over all data samples We use the de-biasing update from MacKay & Ghahramani '05

### CAN WE APPLY THIS TO IMAGE DATA?

### CAN WE ESTIMATE DIMENSIONALITY IN IMAGE DATA?



### CAN WE ESTIMATE DIMENSIONALITY IN IMAGE DATA?



 $\bar{d} = 8$ 





 $\bar{d} = 32$ 



 $\bar{d} = 64$ 



 $\bar{d} = 128$ 

































#### MLE estimates for 10dimensional Basenji's



#### A harder problem: 32-dimensional tree frogs





















































































Higher dimensions is tougher!

32-dimensional tree frogs



## Intrinsic Dimensionality of Common Datasets

(MLE estimates)



# Intrinsic Dimensionality of Common Datasets

(**k=10** MLE estimates)



### IMPACT OF DIMENSIONALITY ON LEARNING

#### Effect of INTRINSIC dimensionality on ResNet-18

Basenji vs Beagle



#### Effect of EXTRINSIC dimensionality on ResNet-18

Basenji vs Beagle



### SUMMARY

- We measured the intrinsic dimensionality of common image datasets.
- We performed a novel large-scale validation of these measurements on GAN generated image data.
- We present novel empirical evidence that the intrinsic but not extrinsic dimension of image datasets matters for generalization with deep networks.

### SUMMARY

More experiments in the paper!

Code available at <a href="https://github.com/ppope/dimensions">https://github.com/ppope/dimensions</a>