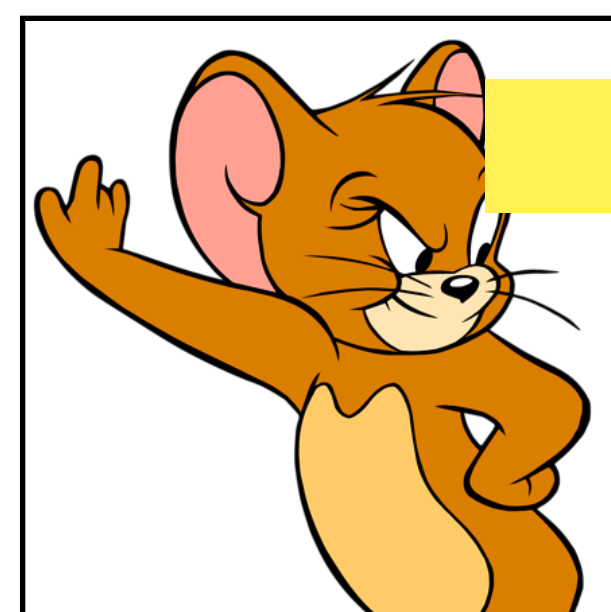




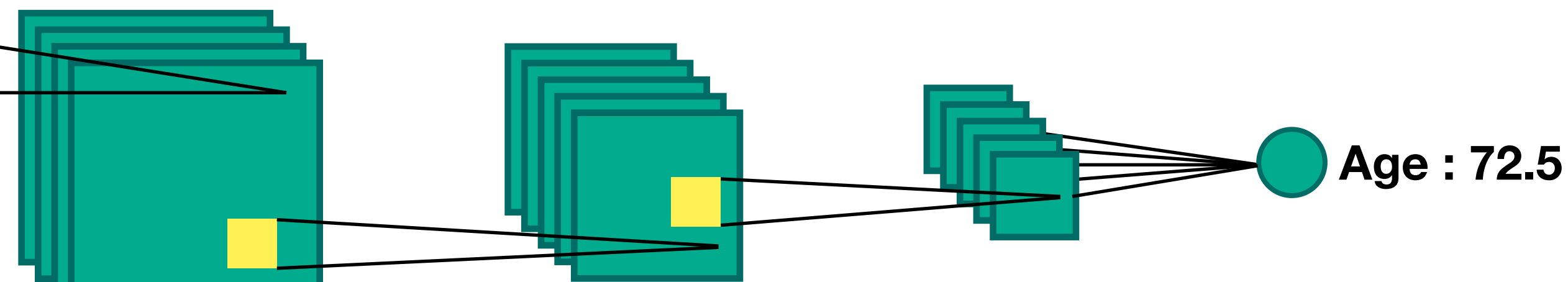
Label Encoding for Regression Networks



<https://medium.com/typeiqs/advanced-lane-finding-c3c8305f074>

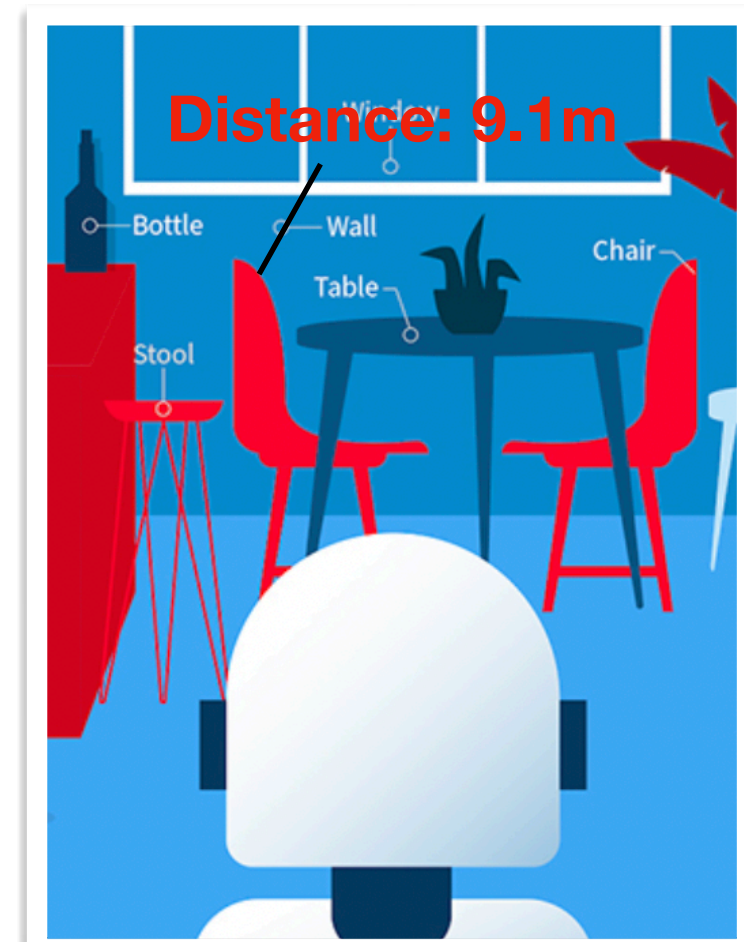


<https://www.pngwing.com/en/free-png-ypjtl>

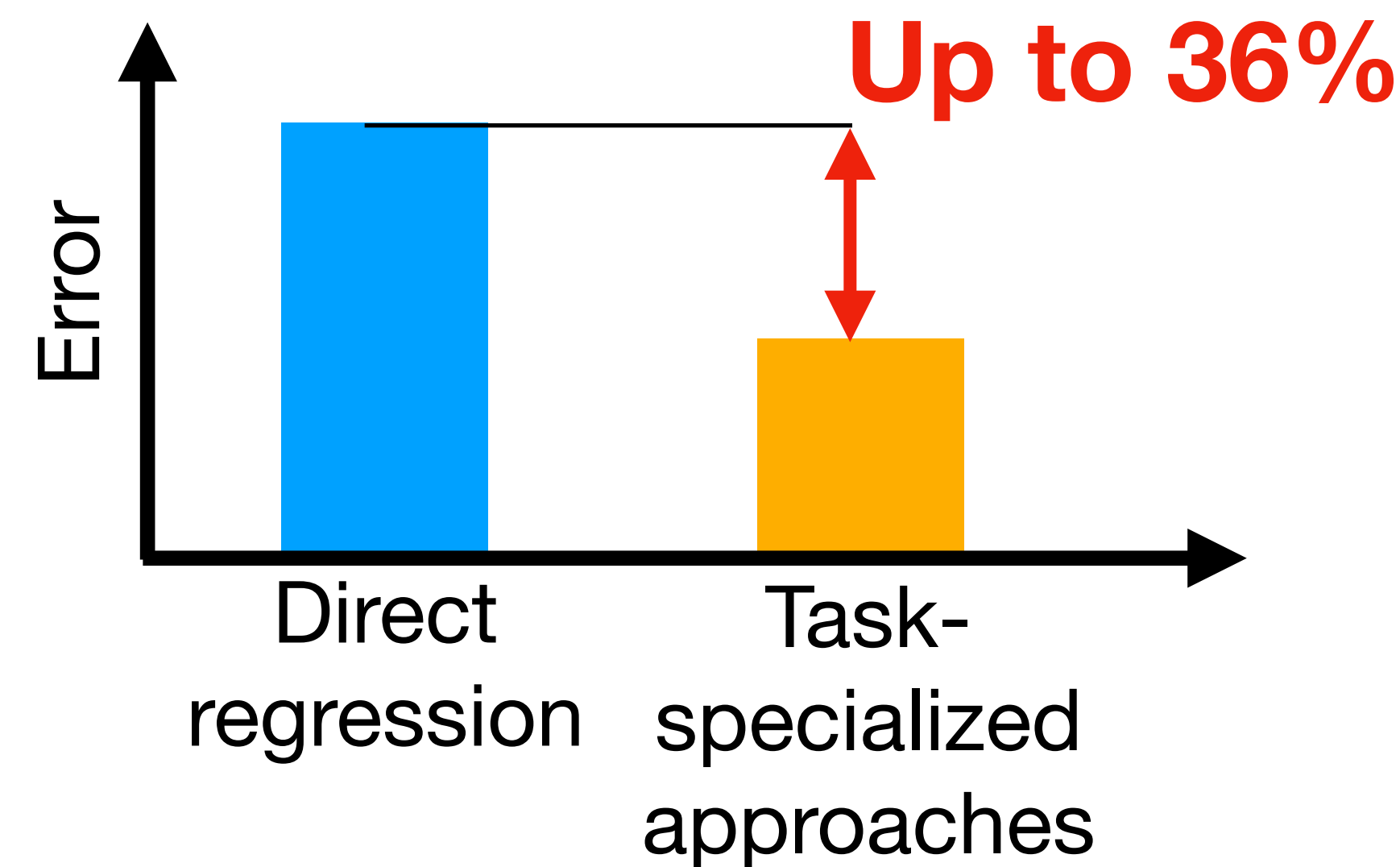


Direct regression:

$$(\text{Predicted value} - \text{Target value})^2$$



<https://www.roboticsbusinessreview.com/opinion/slam-machine-learning-ushers-in-the-age-of-perception/>

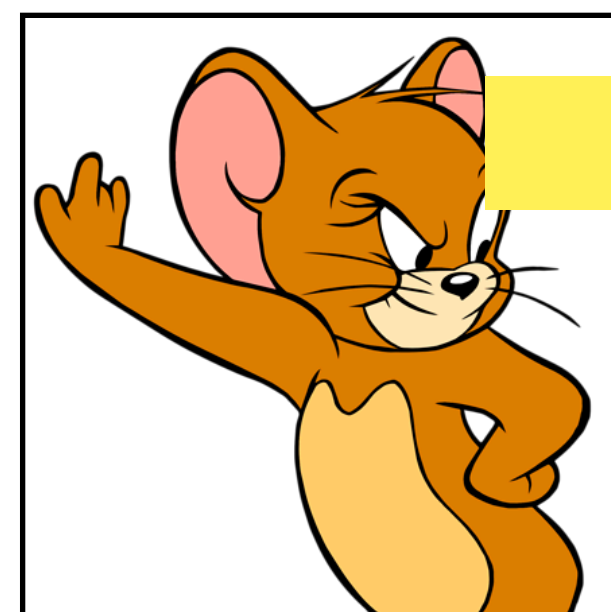




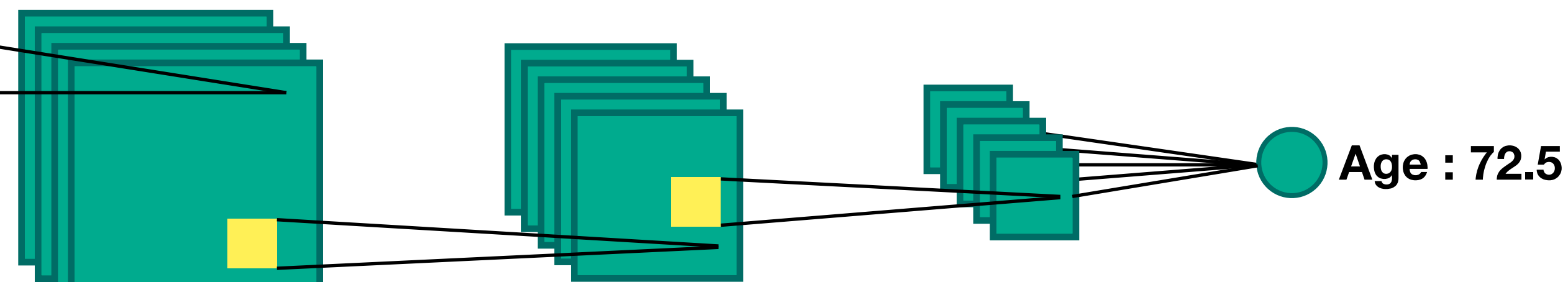
Label Encoding for Regression Networks



<https://medium.com/typeiqs/advanced-lane-finding-c3c8305f074>



<https://www.pngwing.com/en/free-png-ypjtl>



Direct regression:

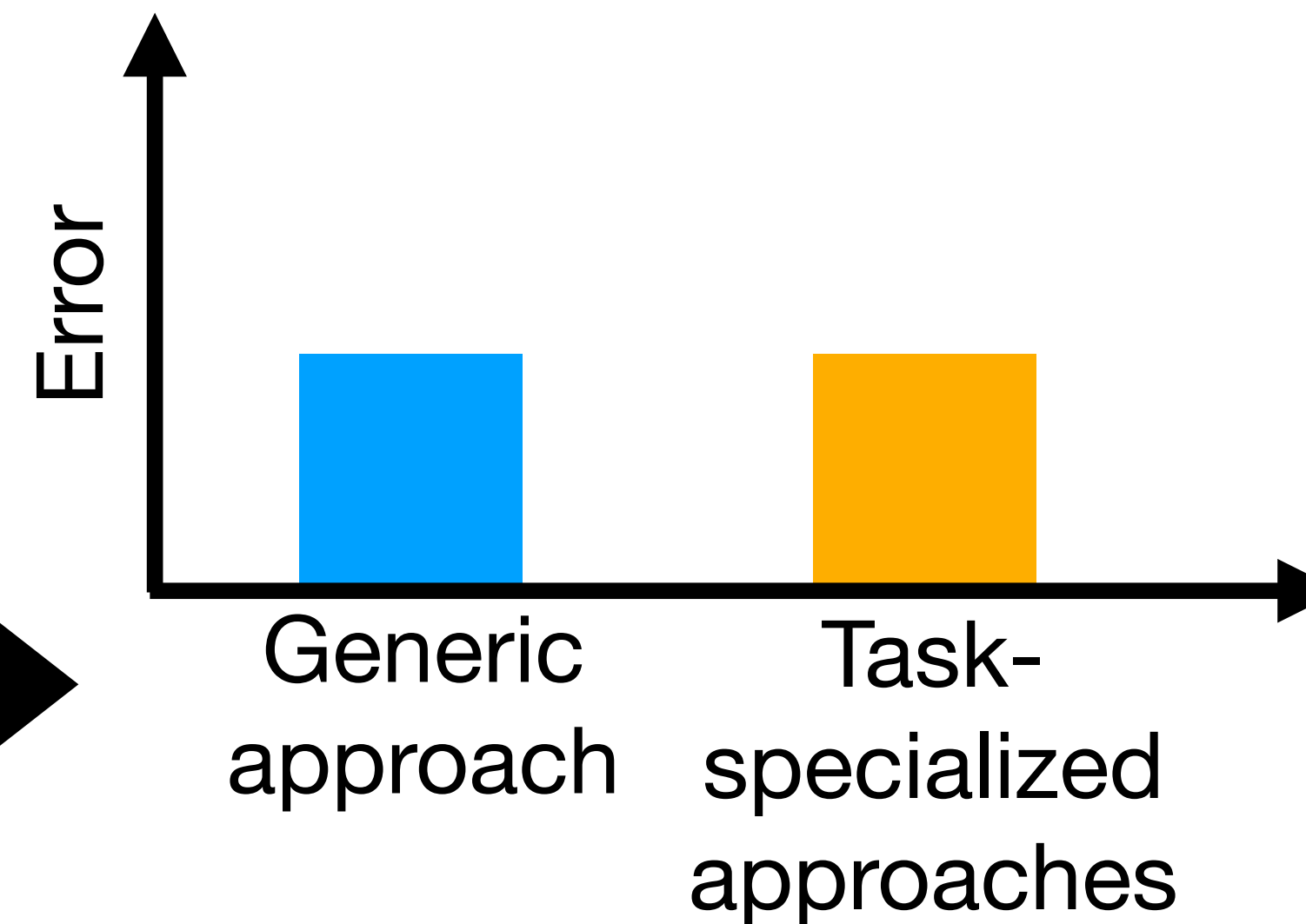
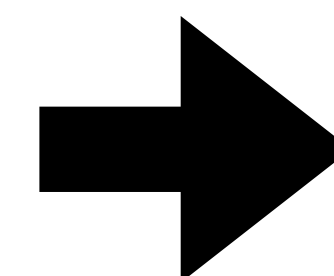
$$(\text{Predicted value} - \text{Target value})^2$$



<https://www.roboticsbusinessreview.com/opinion/slam-machine-learning-ushers-in-the-age-of-perception/>

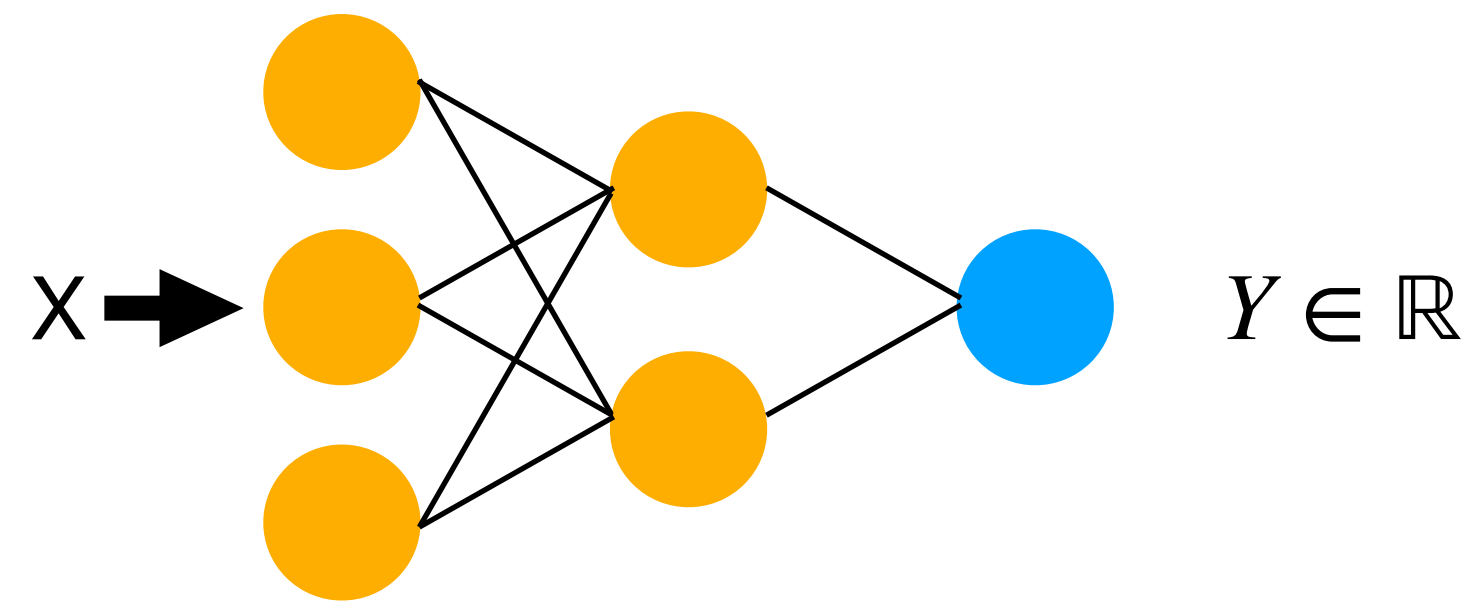


We propose a generic regression approach
“Binary Encoded Labels”



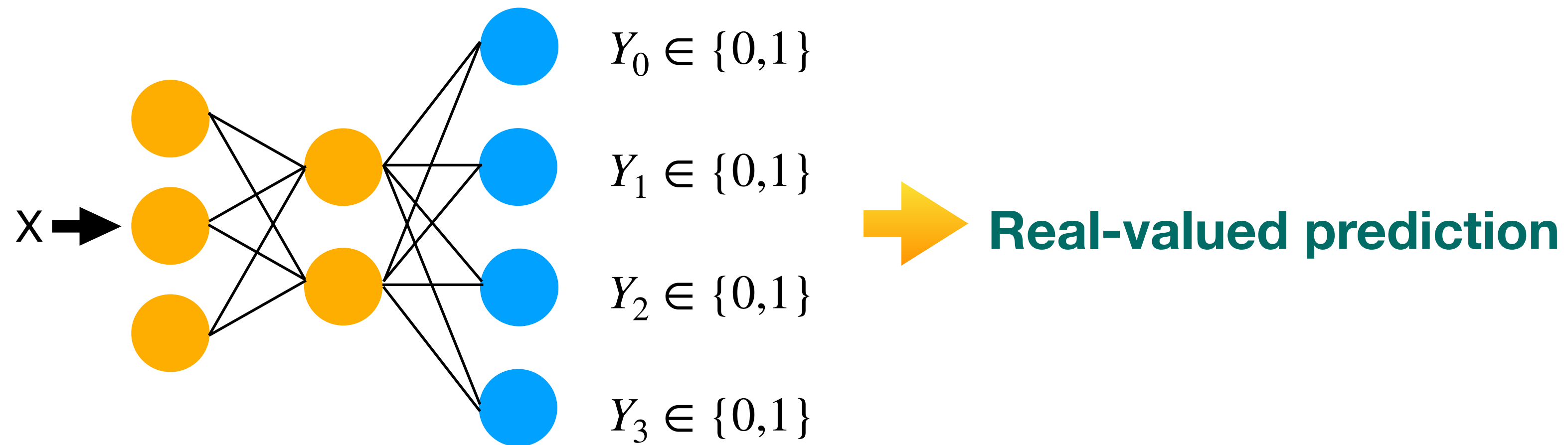
Binary Encoded Labels (BEL)

Regression



Binary Encoded Labels (BEL)

Regression by Binary Classification

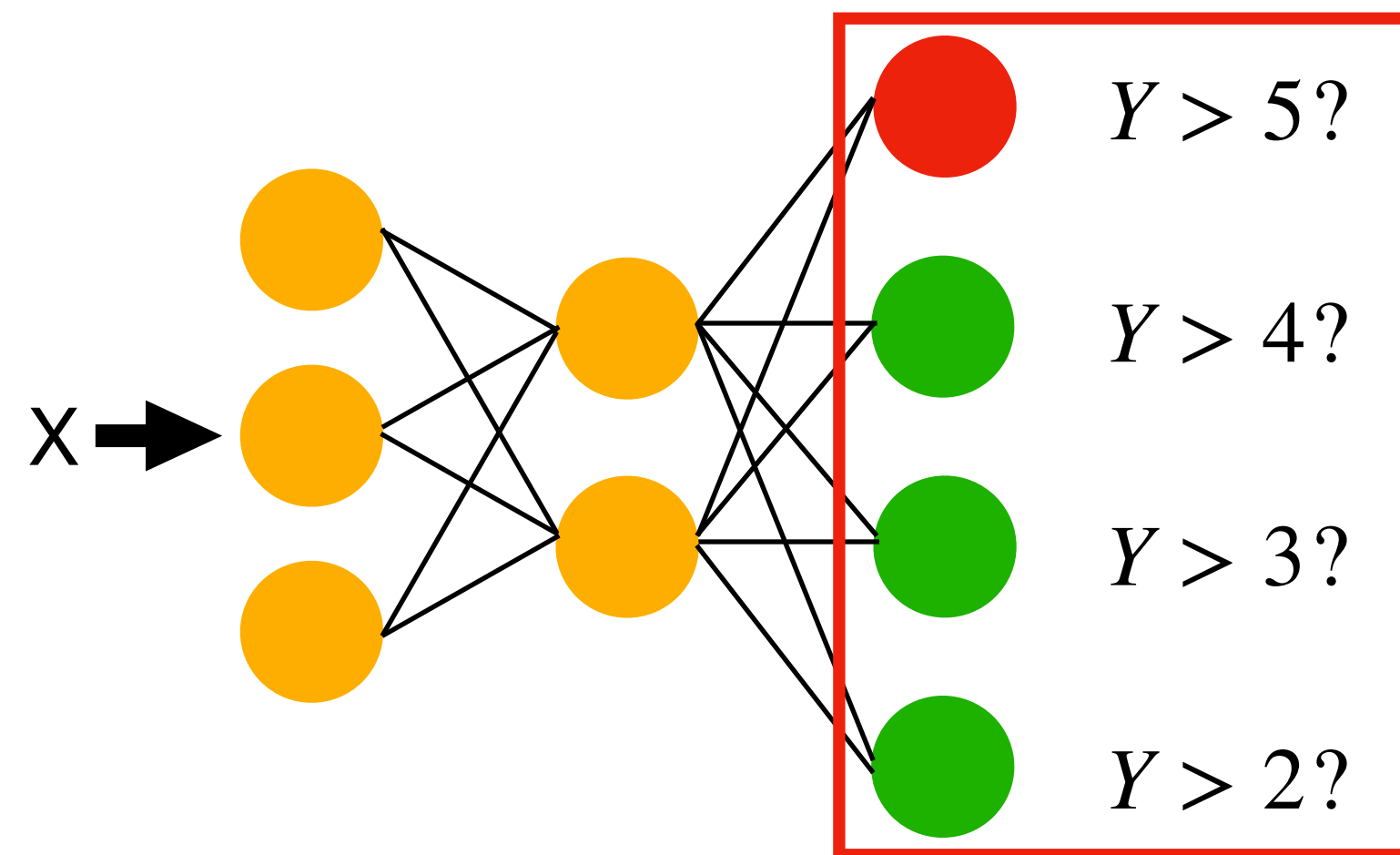


http://www.clipartpanda.com/clipart_images/struggling-uphill-man-with-58696904

Since binary classification is a well-studied problem, **regression by binary classification** can significantly reduce design, implementation, and analysis efforts.

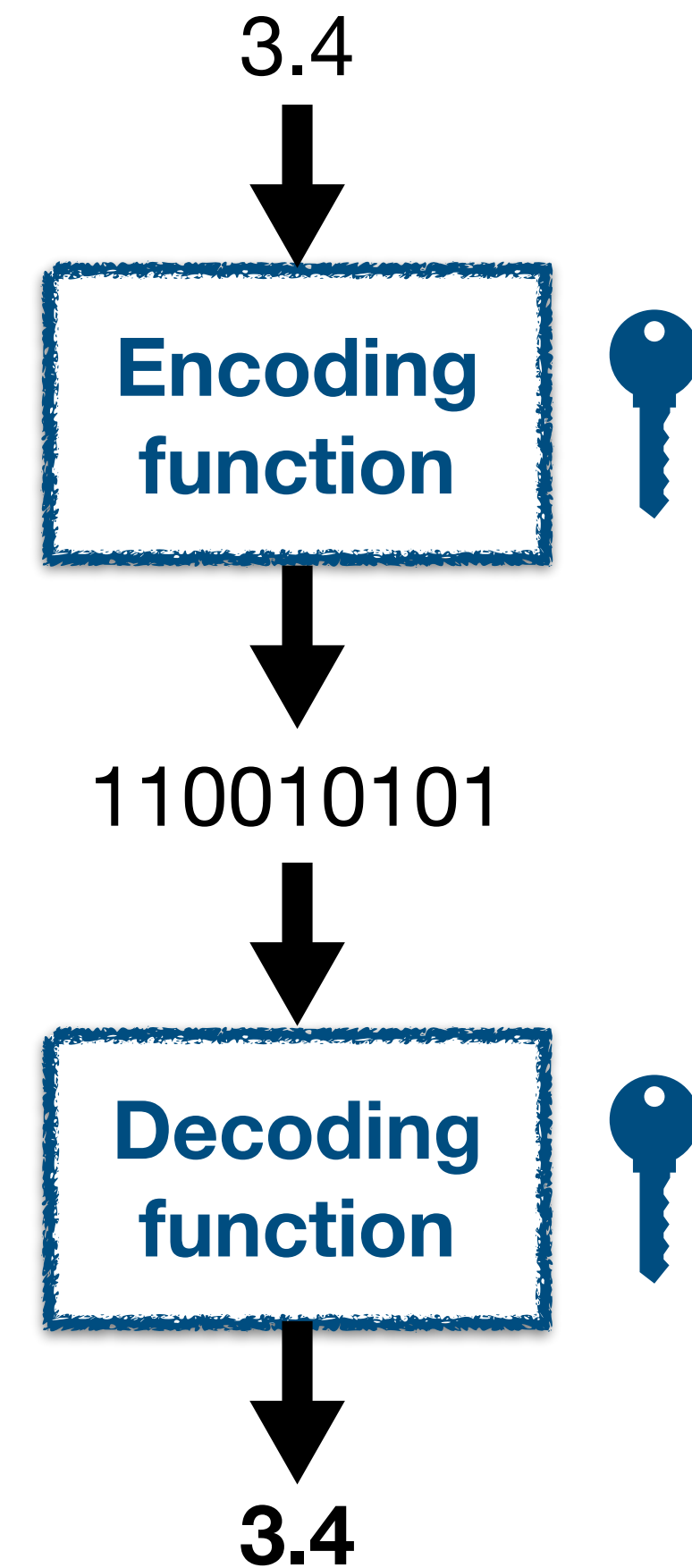
Binary Encoded Labels (BEL)

Setup used by current approaches



$Y \in (2,3]$	$Y \in (3,4]$	$Y \in (4,5]$
0	0	0
0	0	1
0	1	1
1	1	1

Generalization of regression by binary classification (BEL)



[1] Ling Li and Hsuan-Tien Lin. Ordinal regression by extended binary classification, NIPS 2006

[2] Huan Fu, Mingming Gong, Chaohui Wang, Kayhan Batmanghelich, and Dacheng Tao. Deep Ordinal Regression Network for Monocular Depth Estimation, CVPR 2018



Binary Encoded Labels (BEL)

3 values

4 bits

$Y \in (2,3]$	$Y \in (3,4]$	$Y \in (4,5]$

$Y \in (2,3]$	$Y \in (3,4]$	$Y \in (4,5]$

$Y \in (2,3]$	$Y \in (3,4]$	$Y \in (4,5]$

$Y \in (2,3]$	$Y \in (3,4]$	$Y \in (4,5]$

$Y \in (2,3]$	$Y \in (3,4]$	$Y \in (4,5]$

$Y \in (2,3]$	$Y \in (3,4]$	$Y \in (4,5]$



Binary Encoded Labels (BEL)

$Y \in (2,3]$	$Y \in (3,4]$	$Y \in (4,5]$

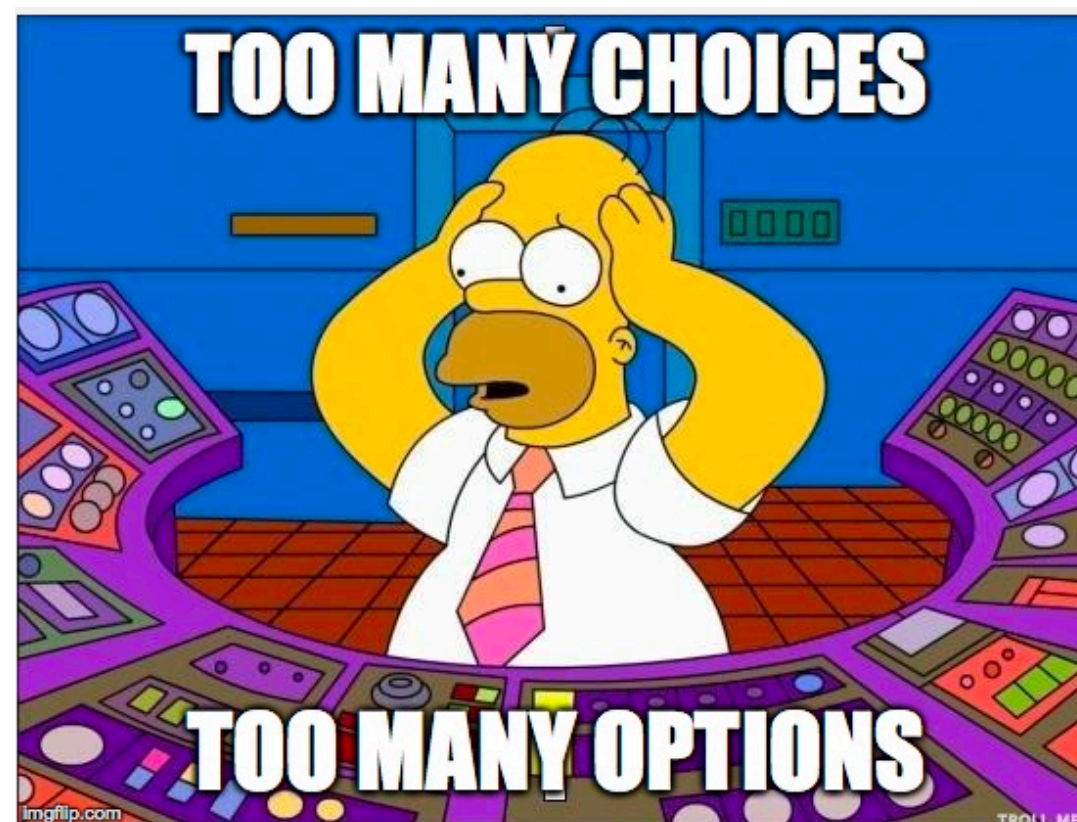
$Y \in (2,3]$	$Y \in (3,4]$	$Y \in (4,5]$

**Number of possible encodings for
M bits
N values**

$$\frac{2^M!}{(2^M - N)!} \rightarrow \frac{2^8!}{(2^8 - 64)!} \rightarrow 10^{89}$$

$Y \in (2,3]$	$Y \in (3,4]$	$Y \in (4,5]$

$Y \in (2,3]$	$Y \in (3,4]$	$Y \in (4,5]$



Binary Encoded Labels (BEL)

- Encoding design is crucial as different codes result in up to **500% higher errors** than empirically found best code.
- However, there has not been a systematic study on this.



<https://www.pngegg.com/en/png-iknaj>

Encoding Design

Label

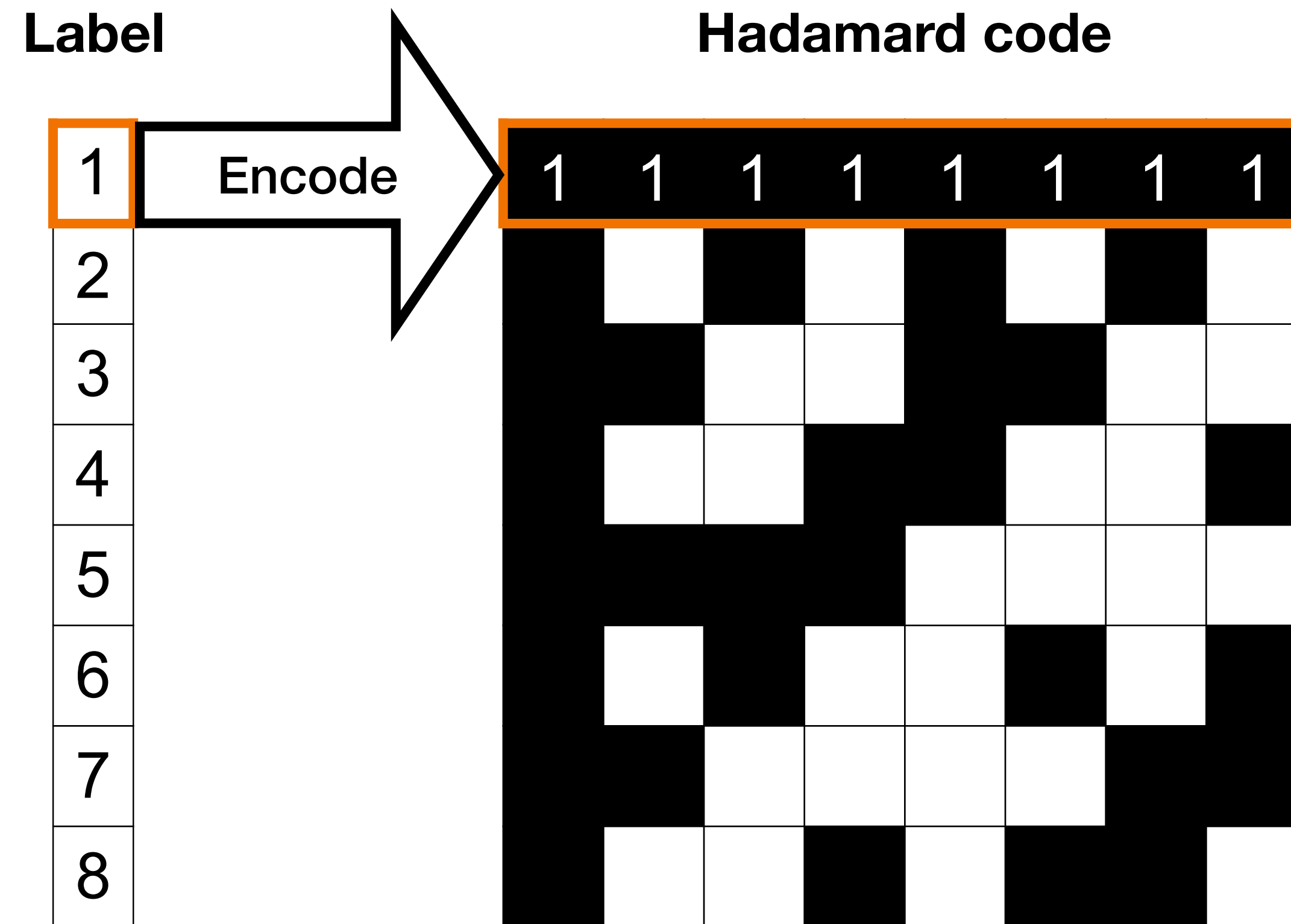
1
2
3
4
5
6
7
8

Error-correction
Hadamard code

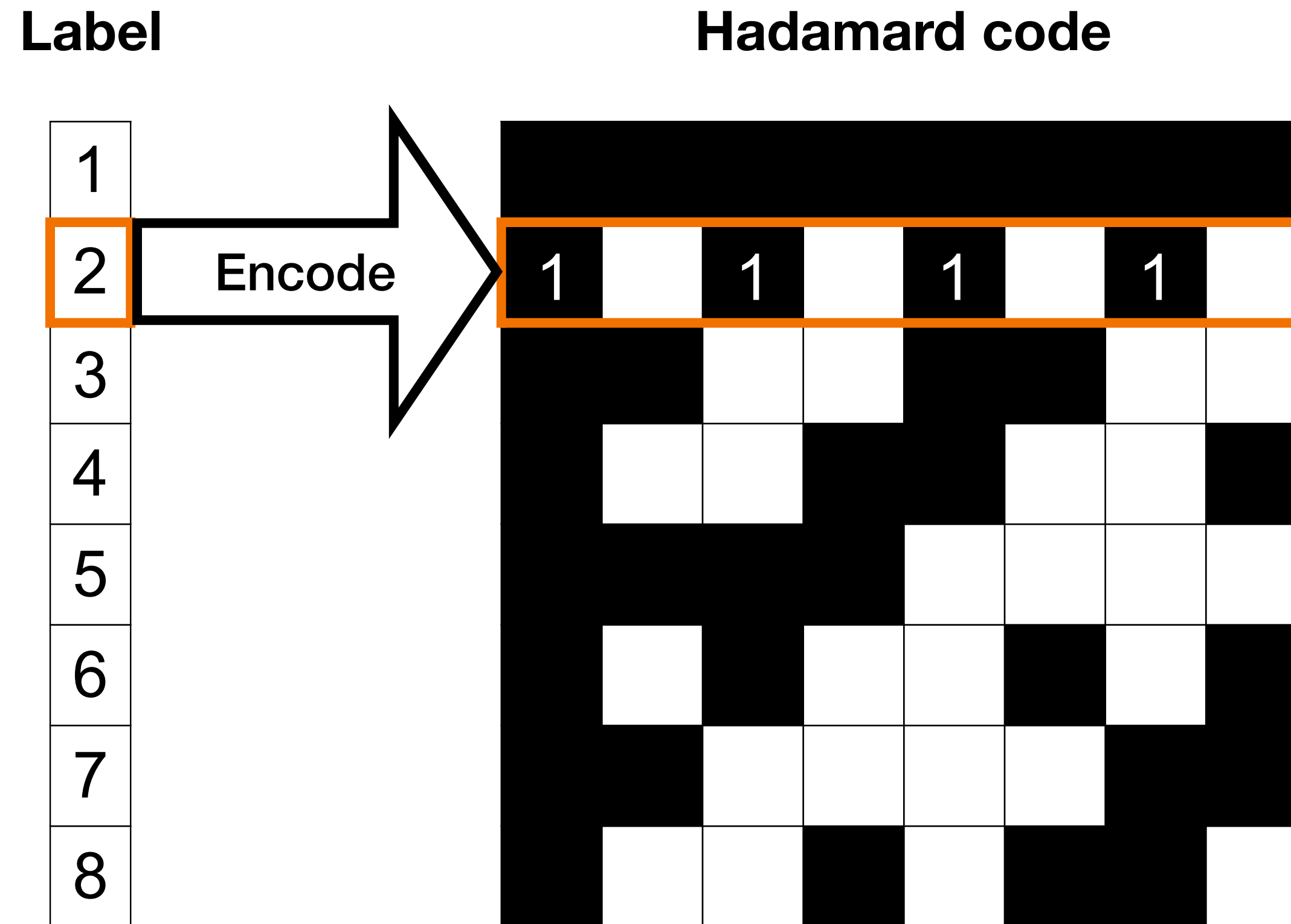
1	1	1	1	1	1	1	1
1	0	1	0	1	0	1	0
1	1	0	0	1	1	0	0
1	0	0	1	1	0	0	1
1	1	1	1	0	0	0	0
1	0	1	0	0	1	0	1
1	1	0	0	0	0	1	1
1	0	0	1	0	1	1	0



Encoding Design

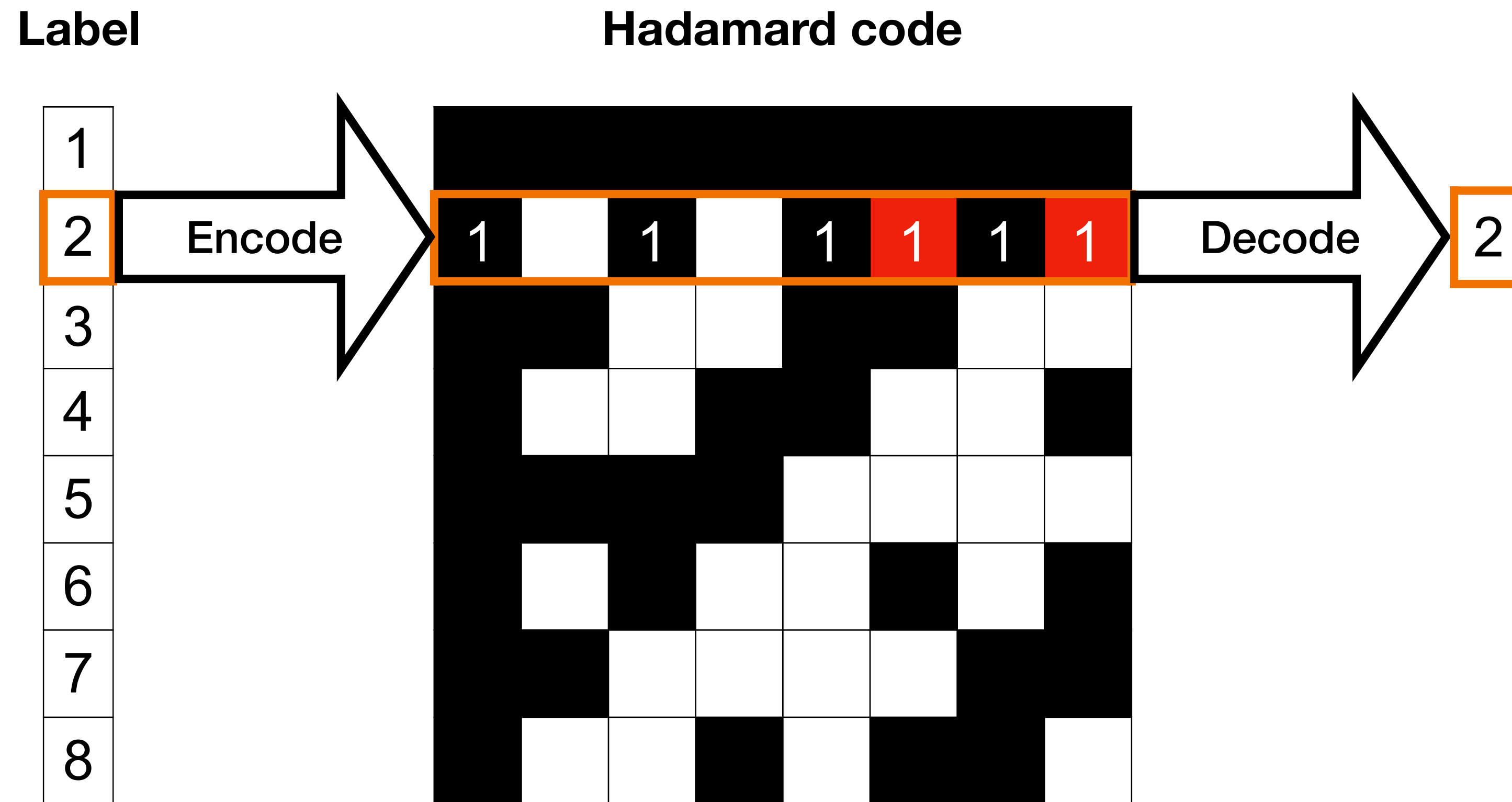


Encoding Design



Encoding Design

- Hadamard code has excellent error-correction properties.
- It is used by existing works on multiclass classification.

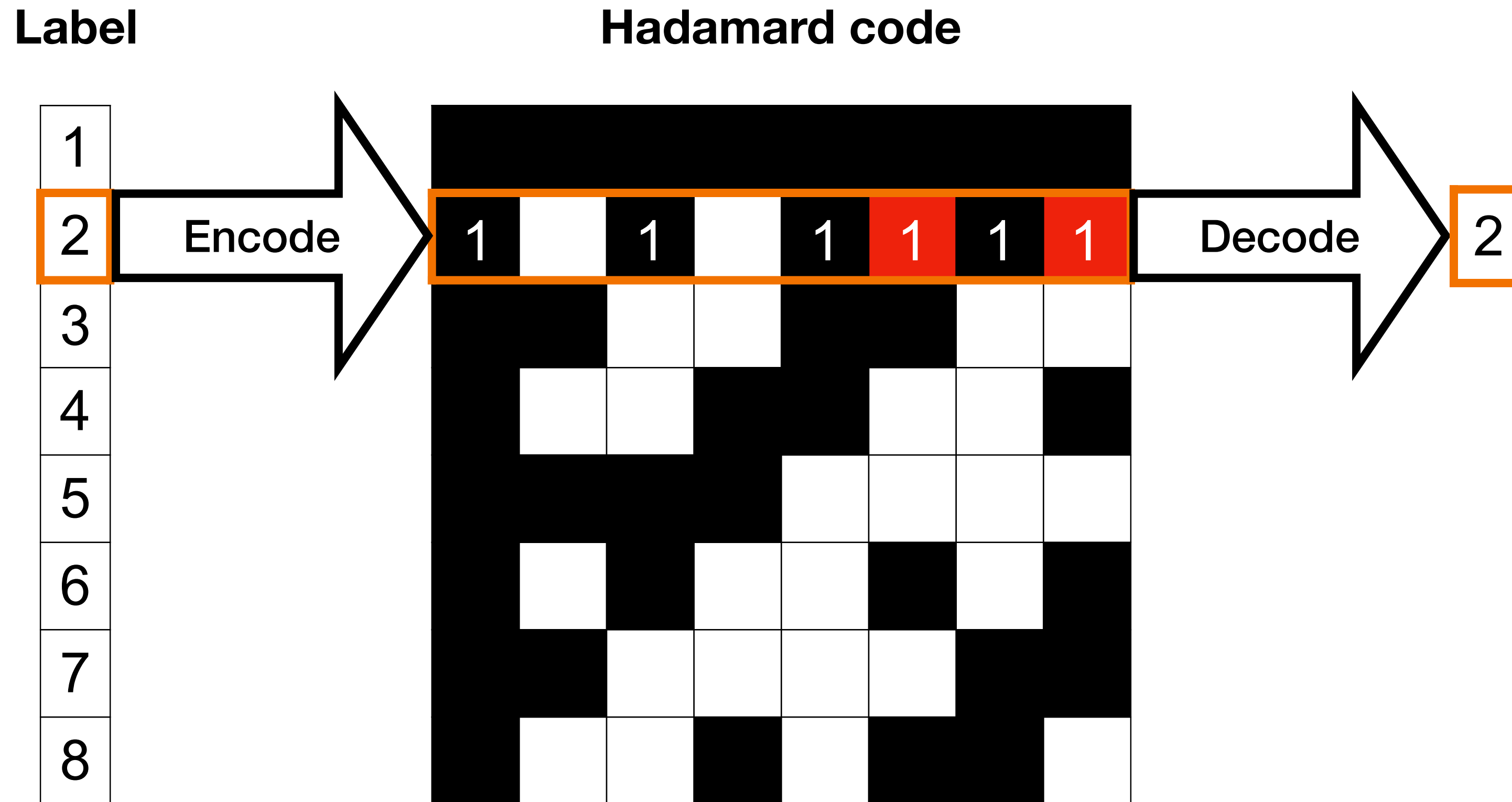
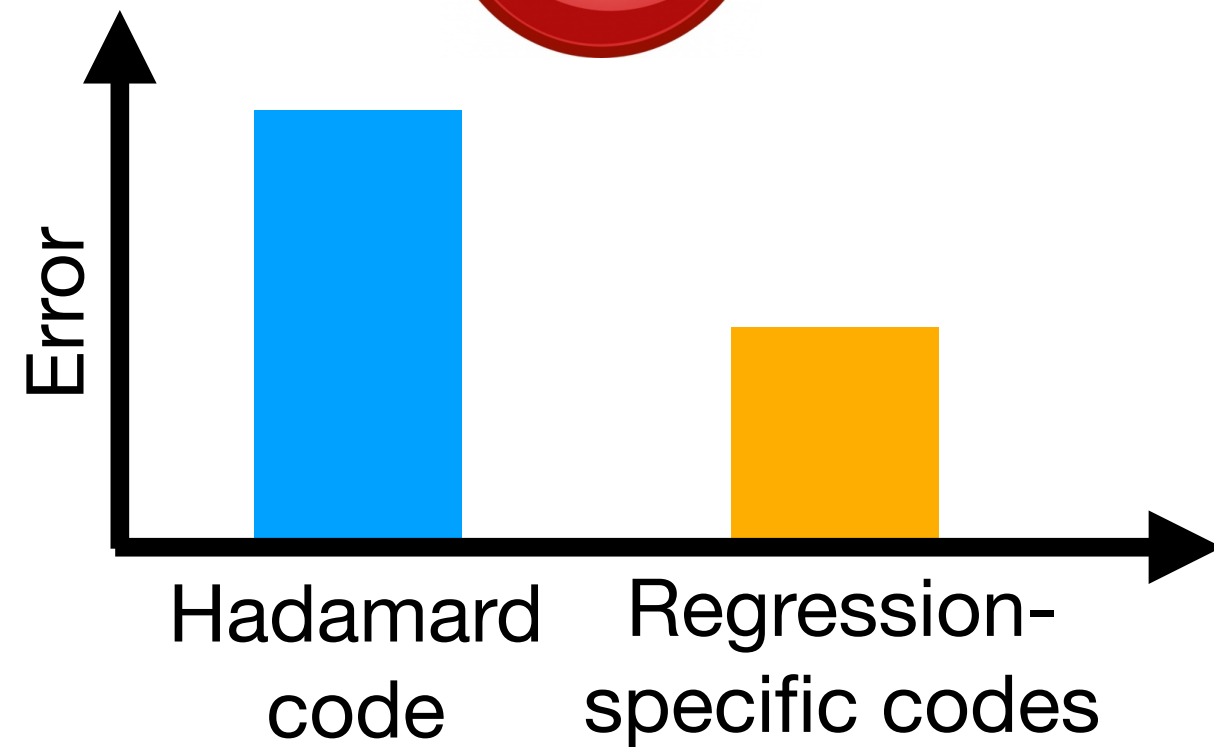


[1] T. G. Dietterich and G. Bakiri. Solving Multiclass Learning Problems via Error-Correcting Output Codes. *Journal of Artificial Intelligence Research*, 1995.

[2] Gunjan Verma and Ananthram Swami. Error correcting output codes improve probability estimation and adversarial robustness of deep neural networks, NeurIPS, 2019.

Encoding Design

We find that the Hadamard code is not suitable for regression!



[1] T. G. Dietterich and G. Bakiri. Solving Multiclass Learning Problems via Error-Correcting Output Codes. *Journal of Artificial Intelligence Research*, 1995.

[2] Gunjan Verma and Ananthram Swami. Error correcting output codes improve probability estimation and adversarial robustness of deep neural networks, NeurIPS, 2019.

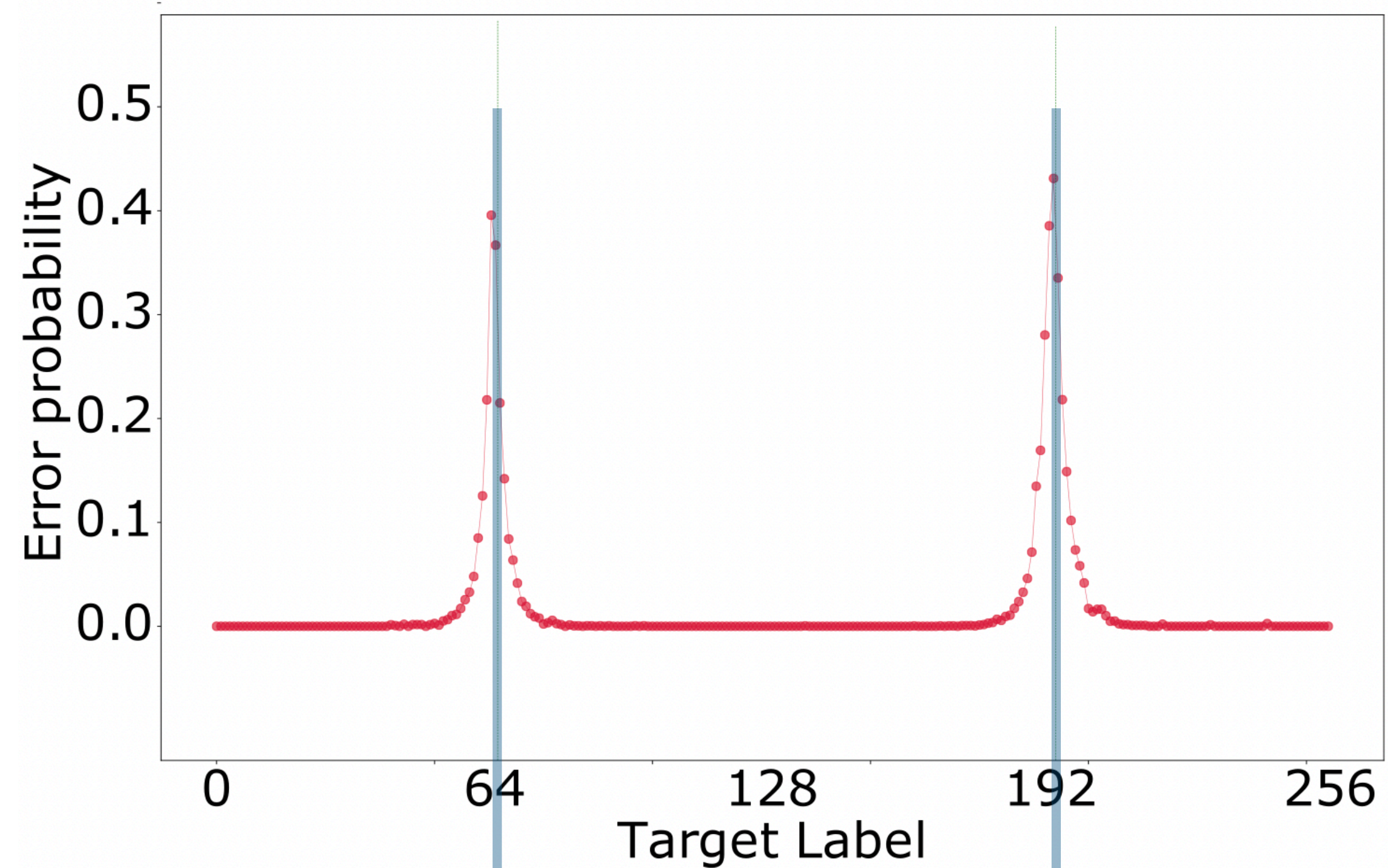


Encoding Design - Key Insight (1)

Target label	0	...	63	64	65	190	191	192	...	255
Classifier's target	0	0	0	1	1	1	1	1	1	0	0	0



Encoding Design - Key Insight (1)

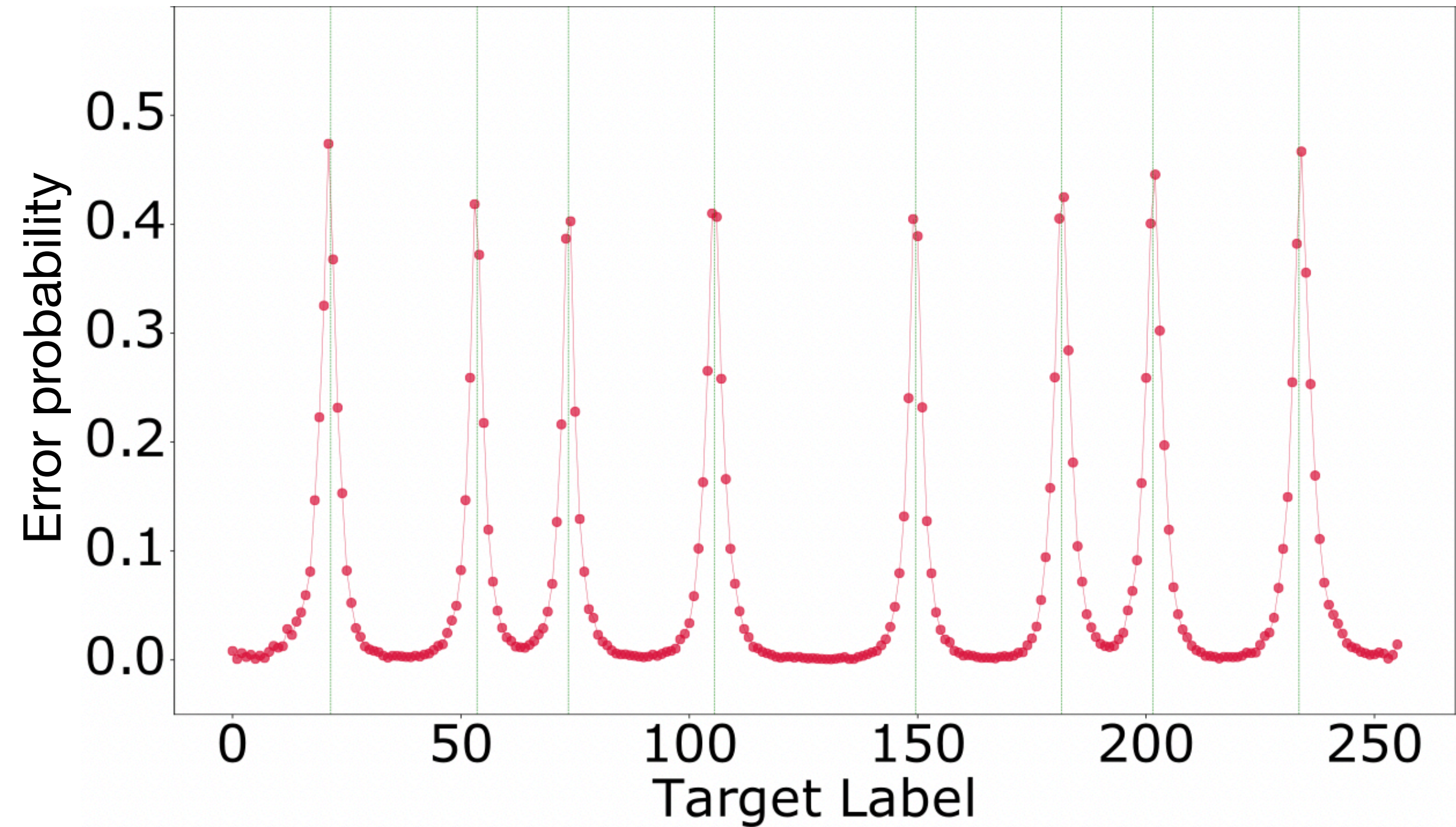


The classifier has a non-uniform error probability for different label values, and it depends on the bit transitions' placement in the code.

Target label	0	...	63	64	65	190	191	192	...	255
Classifier's target	0	0	0	1	1	1	1	1	1	0	0	0



Encoding Design - Key Insight (1)



A classifier's decision boundary becomes more complex with more bit transitions, which degrades its accuracy.

Classifier's target	0..0	11..1	0..0	11..1	00....00	11..1	0..0	11..1	0..0
---------------------	------	-------	------	-------	----------	-------	------	-------	------



Encoding Design - Key Insight (1)

Bit-error probability



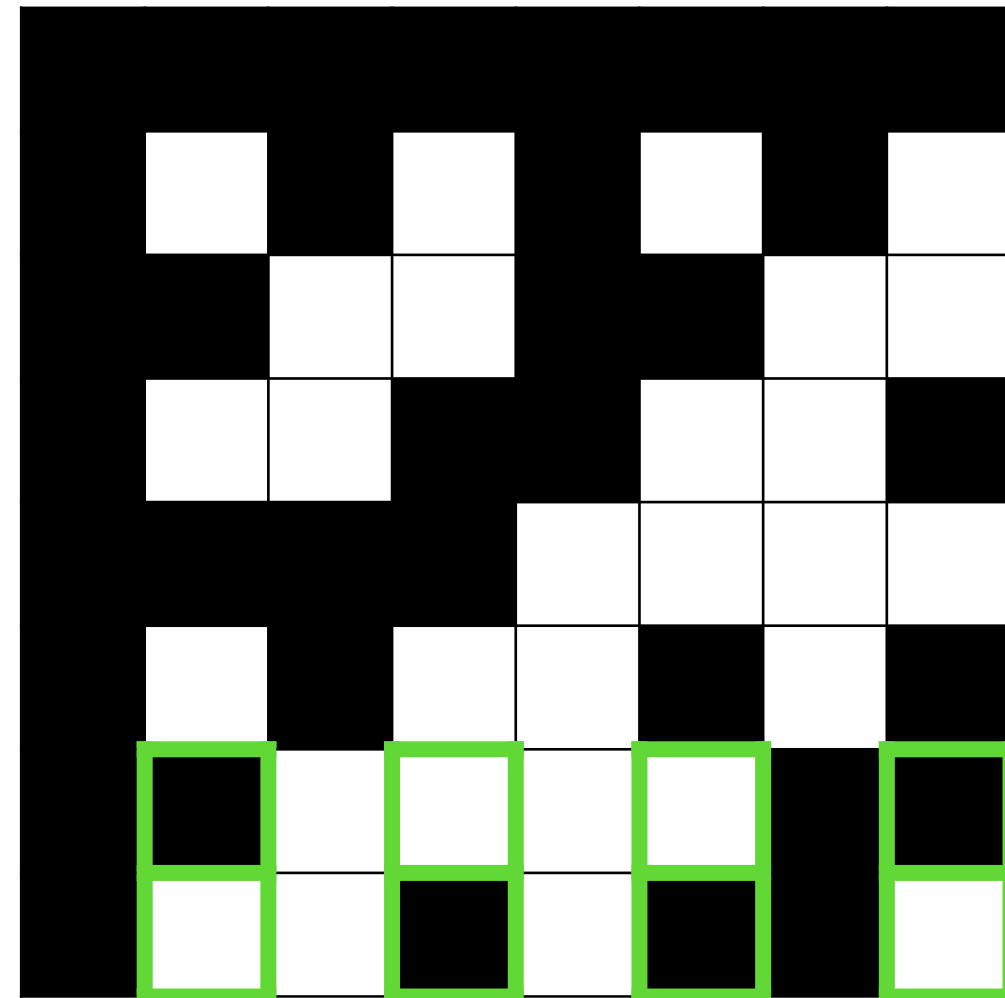
Error-correction capability

Hamming Distance

Label

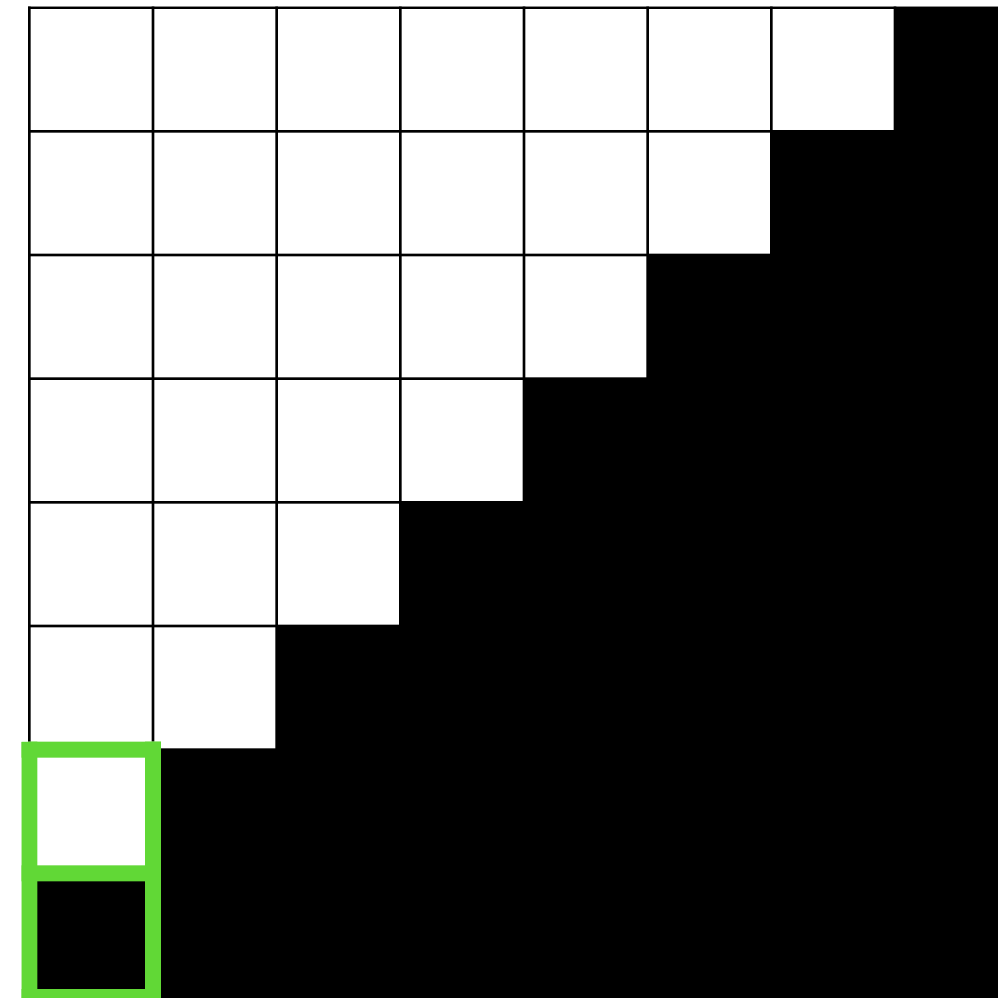
1
2
3
4
5
6
7
8

Hadamard code



4

Unary code



1



Encoding Design - Key Insight (1)

Bit-error probability



Error-correction capability

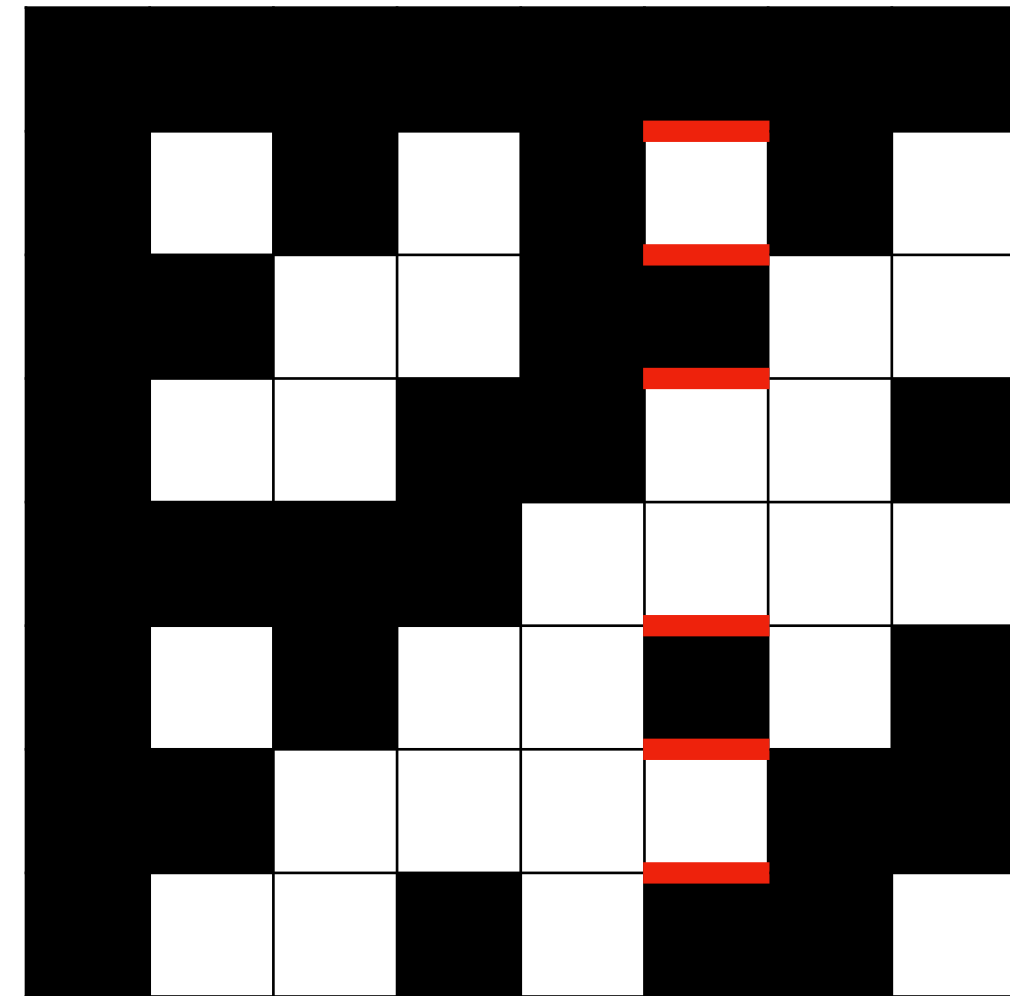
Hamming Distance

Classification Error

Label

1
2
3
4
5
6
7
8

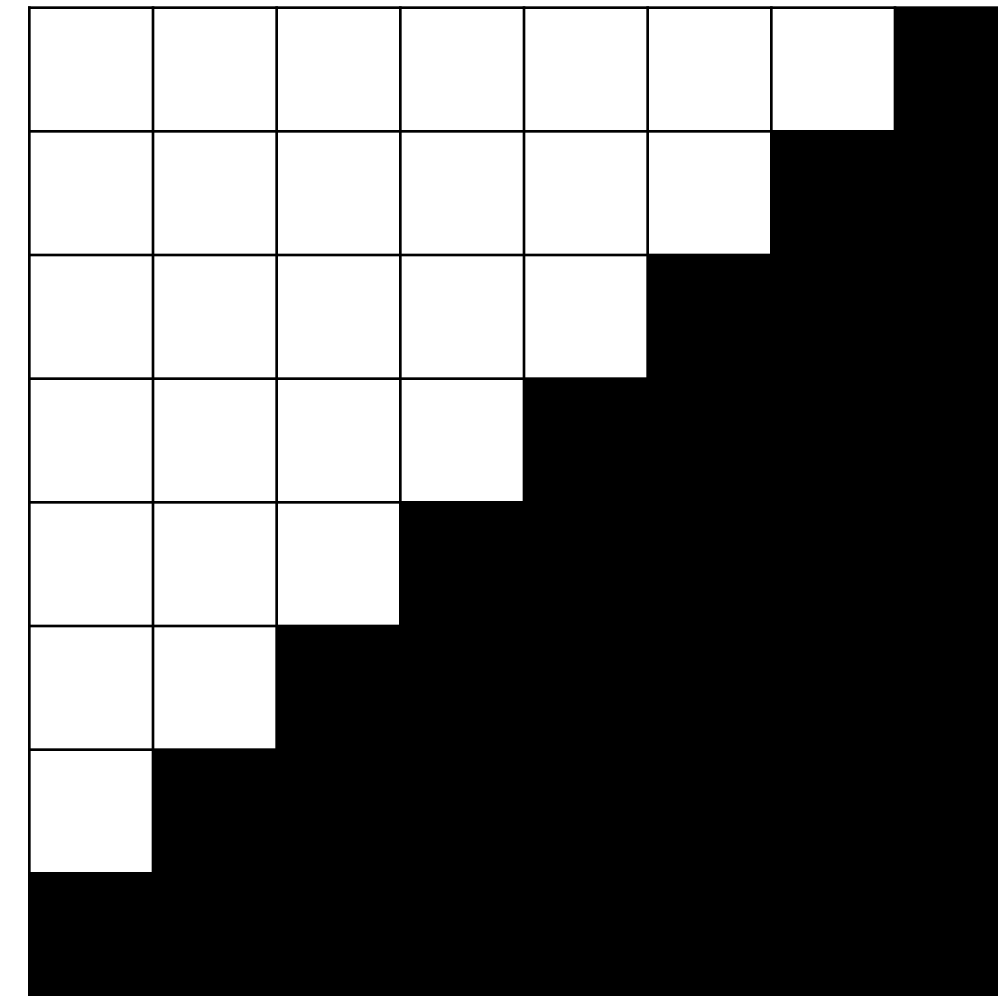
Hadamard code



4

~50%

Unary code



1

~2%



Encoding Design - Key Insight (2)

Regression objective :  | Predicted value - Target value |

Label Hadamard code

1	0	0	0	0	0	0	0
2	1	0	1	0	1	0	1
3	0	1	1	0	0	1	0
4	1	1	0	0	1	0	1
5	0	0	1	1	0	1	0
6	1	0	1	0	1	0	1
7	0	1	0	1	0	1	0
8	1	1	0	1	0	1	0

Label Unary code

1	0	0	0	0	0	0	1
2	0	0	0	0	0	1	1
3	0	0	0	0	1	1	1
4	0	0	0	1	1	1	1
5	0	0	1	1	1	1	1
6	0	1	1	1	1	1	1
7	1	1	1	1	1	1	1
8	1	1	1	1	1	1	1

Encoding Design - Key Insight (2)

Regression objective :  $| \text{Predicted value} - \text{Target value} |$

The hamming distance between two codes should increase with the difference between corresponding label values.

Label **Hadamard code**

1	Black	Black	Black	Black	Black	Black	Black
2	White	Black	White	Black	White	Black	White
3	Black	White	Black	White	Black	White	Black
4	White	White	Black	Black	White	White	Black
5	Black	Black	Black	White	White	White	White
6	White	Black	White	White	Black	White	Black
7	Black	White	Black	White	Black	White	Black
8	White	White	Black	White	Black	White	Black

Label **Unary code**

1	White	White	White	White	White	White	White	Black
2	White	White	White	White	White	White	Black	Black
3	White	White	White	White	Black	Black	Black	Black
4	White	White	White	Black	Black	Black	Black	Black
5	White	White	Black	Black	Black	Black	Black	Black
6	White	Black	Black	Black	Black	Black	Black	Black
7	Black	Black	Black	Black	Black	Black	Black	Black
8	Black	Black	Black	Black	Black	Black	Black	Black

Hamming distance : 1

Encoding Design - Key Insight (2)

Regression objective :  $| \text{Predicted value} - \text{Target value} |$

The hamming distance between two codes should increase with the difference between corresponding label values.

Label **Hadamard code**

1	[All black]					
2	White	Black	White	Black	White	Black
3	Black	White	Black	White	Black	White
4	White	White	Black	Black	White	Black
5	Black	Black	White	White	Black	White
6	White	Black	White	Black	White	Black
7	Black	White	Black	White	Black	White
8	White	White	Black	Black	White	Black

Label **Unary code**

1	White	White	White	White	White	White	White	Black
2	White	White	White	White	White	White	Black	Black
3	White	White	White	White	White	Black	Black	Black
4	White	White	White	Black	Black	Black	Black	Black
5	White	White	Black	Black	Black	Black	Black	Black
6	White	Black	Black	Black	Black	Black	Black	Black
7	Black	Black	Black	Black	Black	Black	Black	Black
8	Black	Black	Black	Black	Black	Black	Black	Black

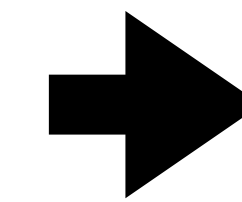
Hamming distance : 2



Encoding Design - Key Insight (2)

Regression objective :  $|\text{Predicted value} - \text{Target value}|$

The hamming distance between two codes should increase with the difference between corresponding label values.



Reduces the probability of making large regression errors.

Label **Hadamard code**

1	Black	Black	Black	Black	Black	Black	Black
2	White	Black	White	Black	White	Black	White
3	Black	White	Black	White	Black	White	Black
4	White	White	Black	Black	White	White	Black
5	Black	Black	Black	White	White	White	White
6	White	Black	White	White	Black	White	Black
7	Black	White	Black	White	Black	White	Black
8	White	White	Black	White	Black	White	Black

Label **Unary code**

1	White	White	White	White	White	White	White	Black
2	White	White	White	White	White	White	Black	Black
3	White	White	White	White	White	Black	Black	Black
4	White	White	White	White	Black	Black	Black	Black
5	White	White	Black	Black	Black	Black	Black	Black
6	White	Black	Black	Black	Black	Black	Black	Black
7	Black	Black	Black	Black	Black	Black	Black	Black
8	Black	Black	Black	Black	Black	Black	Black	Black

Hamming distance : 3

Unary code follows this property and results in lower regression error than Hadamard code.



Encoding Design - Key Insight (3)

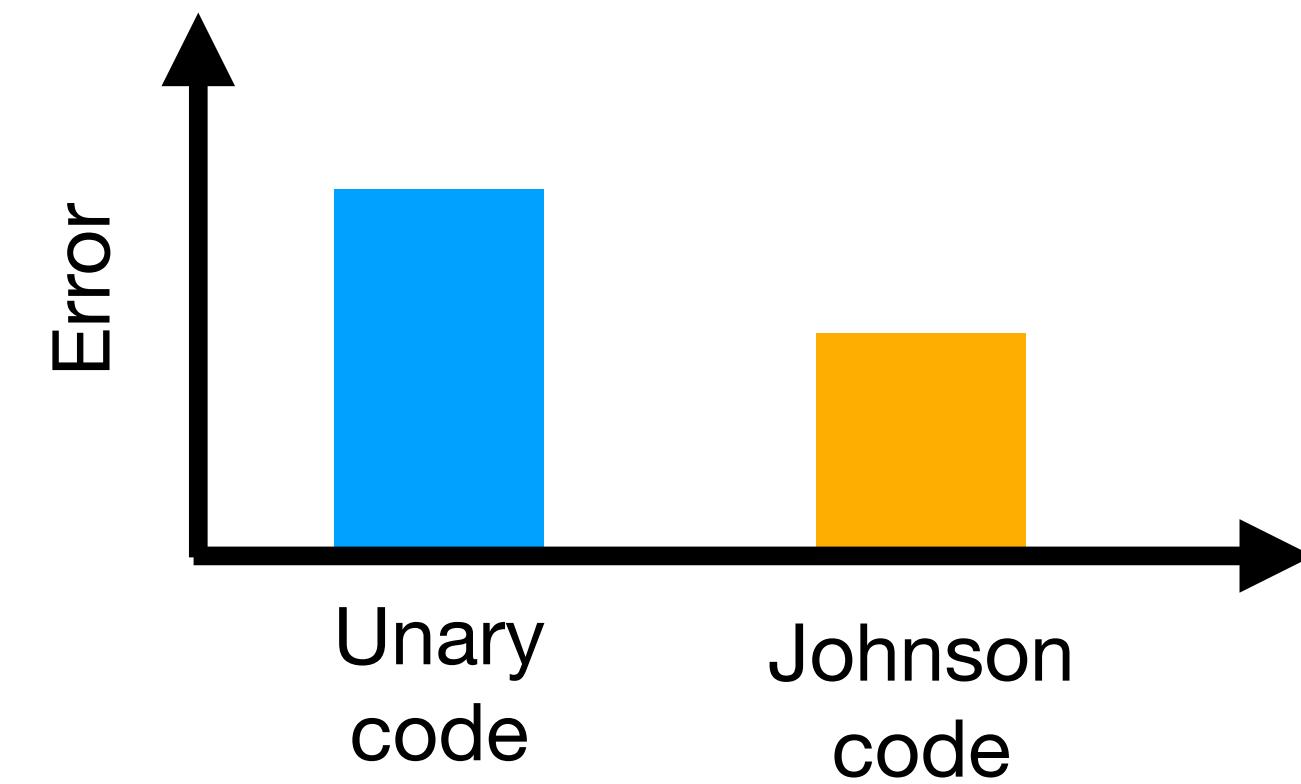
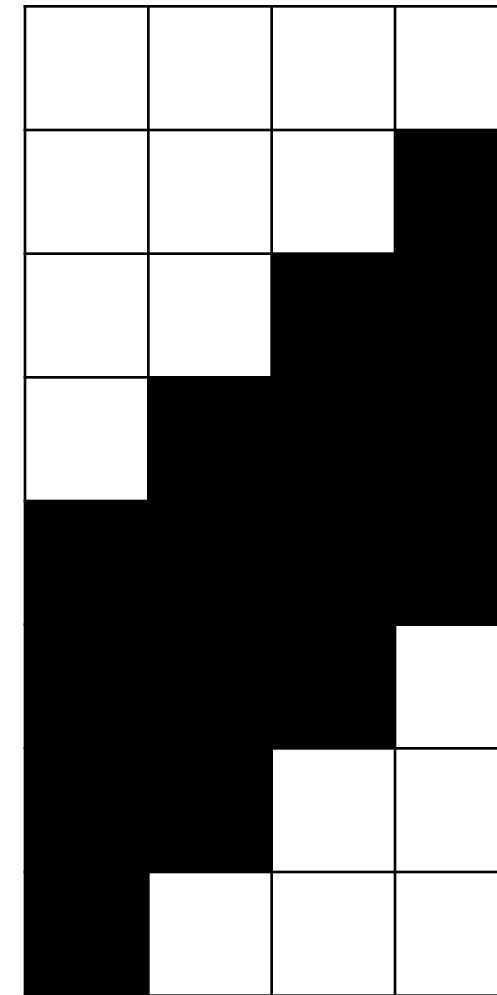
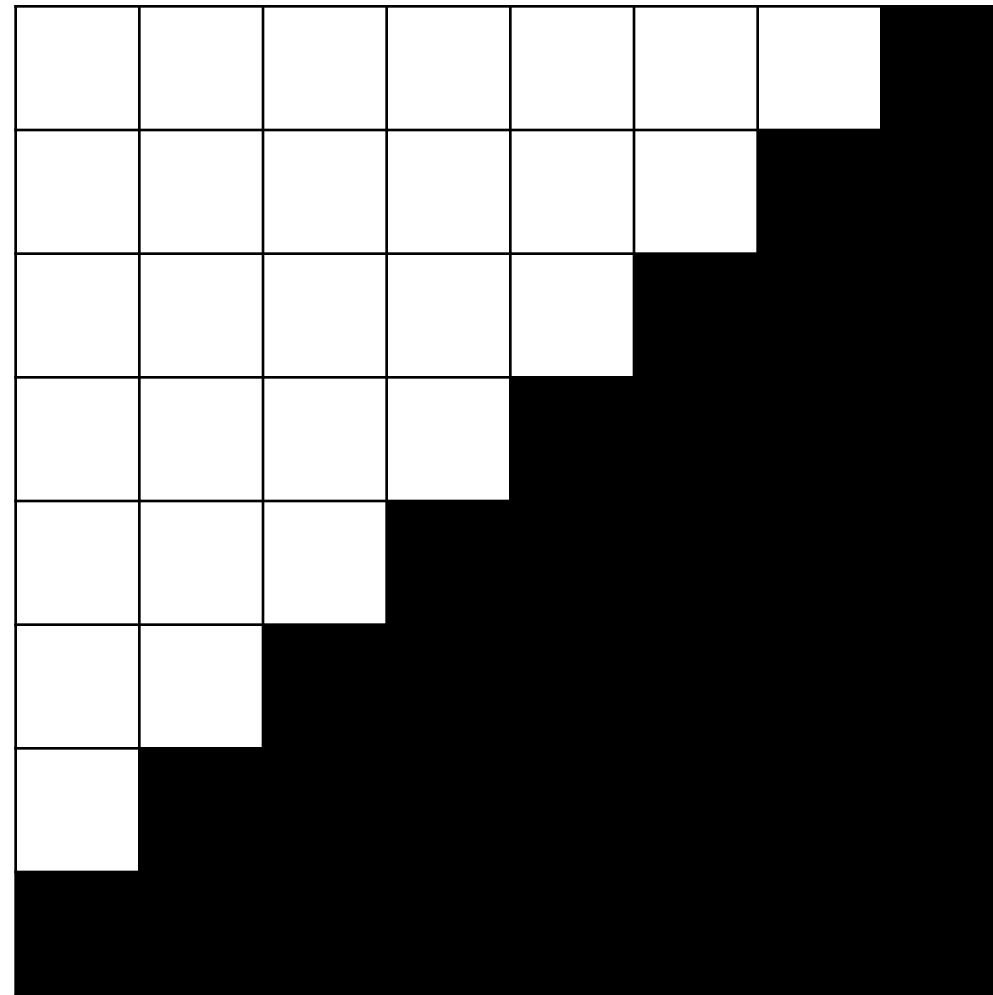
Classifiers' nonuniform error probability distribution can be exploited to design better and more compact codes.

Label

Unary code

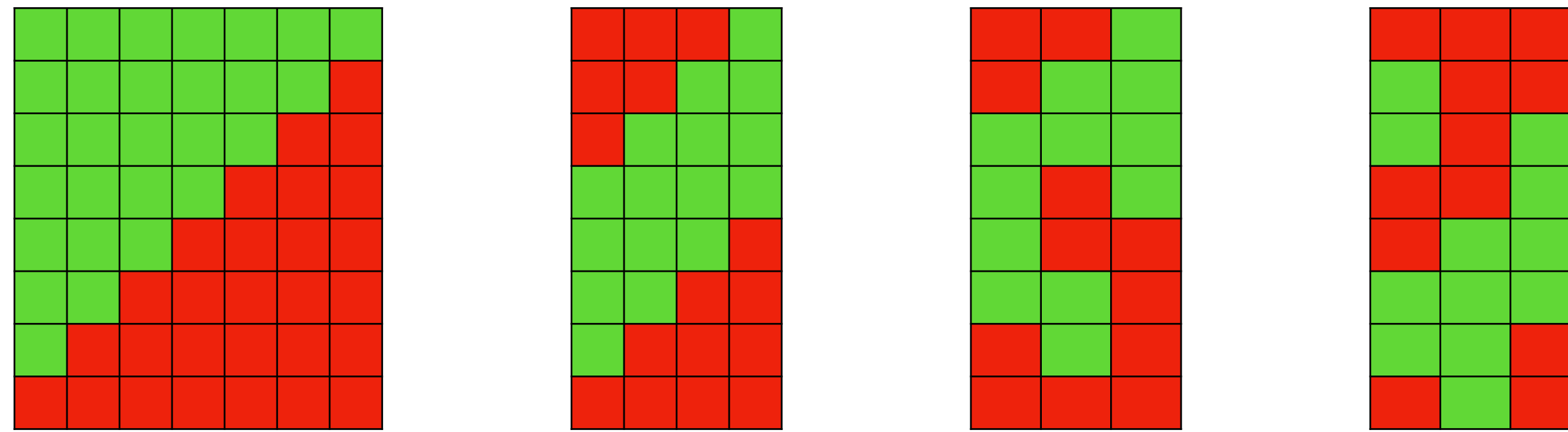
Johnson code

1
2
3
4
5
6
7
8

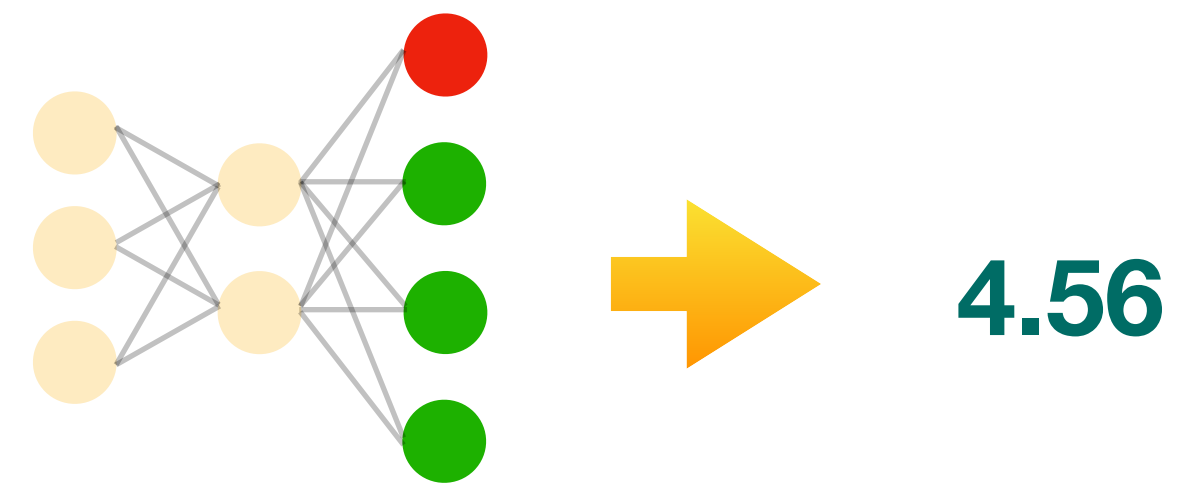


BEL Design Parameters

- Encoding functions



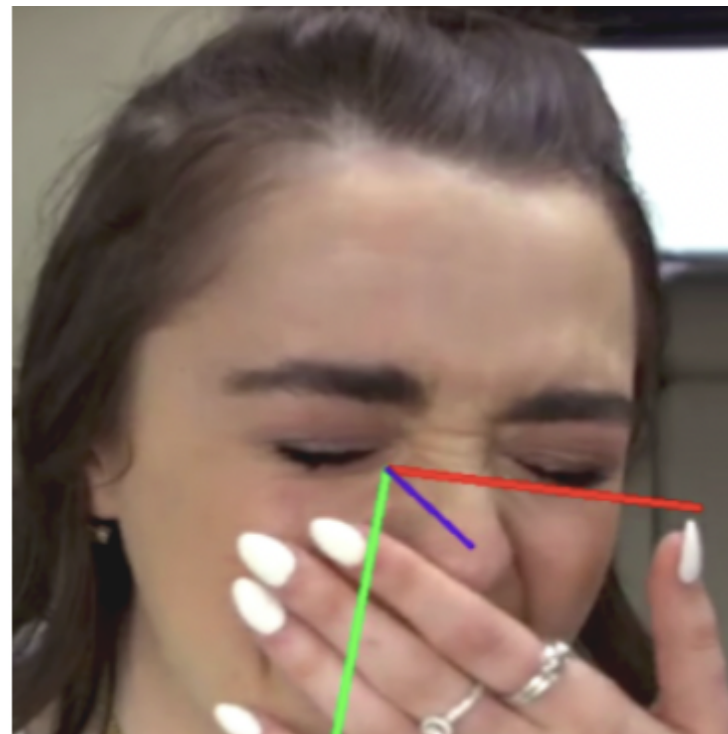
- Decoding functions



- Loss functions

$$\mathcal{L}(Y, \hat{Y})$$

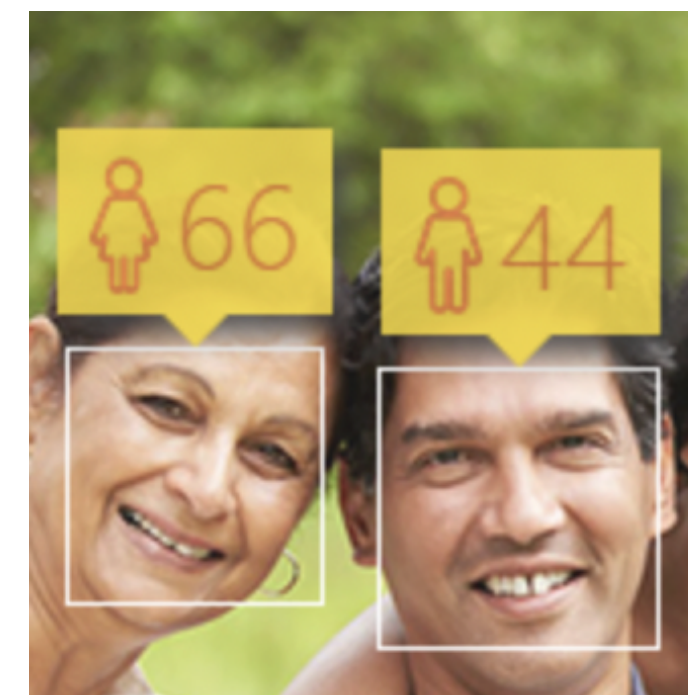
Evaluation



Head pose estimation



Facial landmark detection



Age estimation



End-to-end autonomous driving

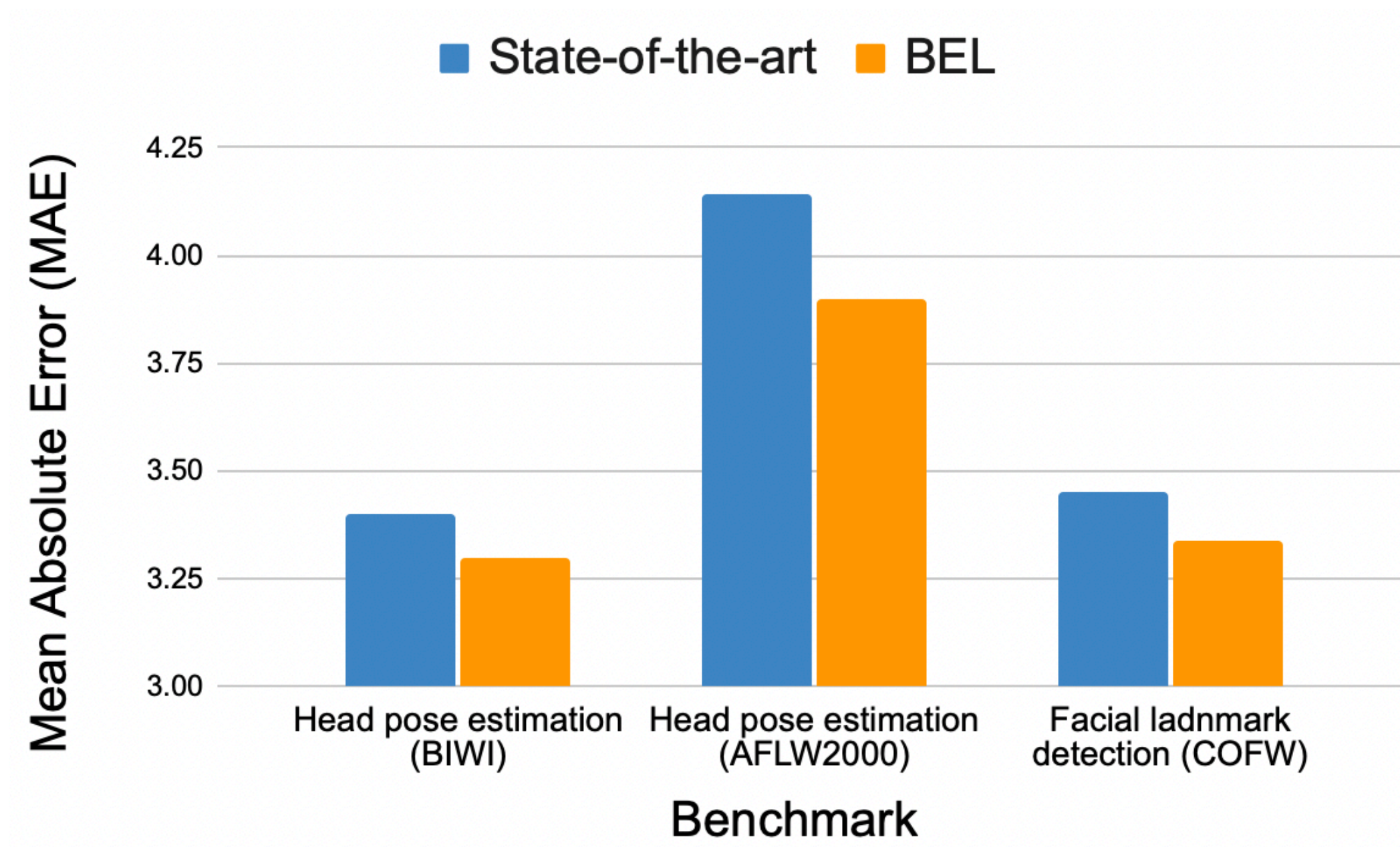
Image sources:

1. <https://github.com/natanielruiz/deep-head-pose>
2. D. Merget, M. Rock and G. Rigoll, "Robust Facial Landmark Detection via a Fully-Convolutional Local-Global Context Network," *CVPR* 2018
3. <https://techxplore.com/news/2015-05-microsoft-age-estimate-tool-unleashed-real-time.html>
4. <https://www.idtechex.com/en/research-article/how-close-are-we-to-autonomous-cars/19191>

Evaluation

10% lower error than direct regression

7.2% lower error than specialized approaches



Check our paper for more information!

https://github.com/ubc-aamodt-group/BEL_regression

