SINGER: Stochastic Network Graph Evolving Operator for High Dimensional PDEs

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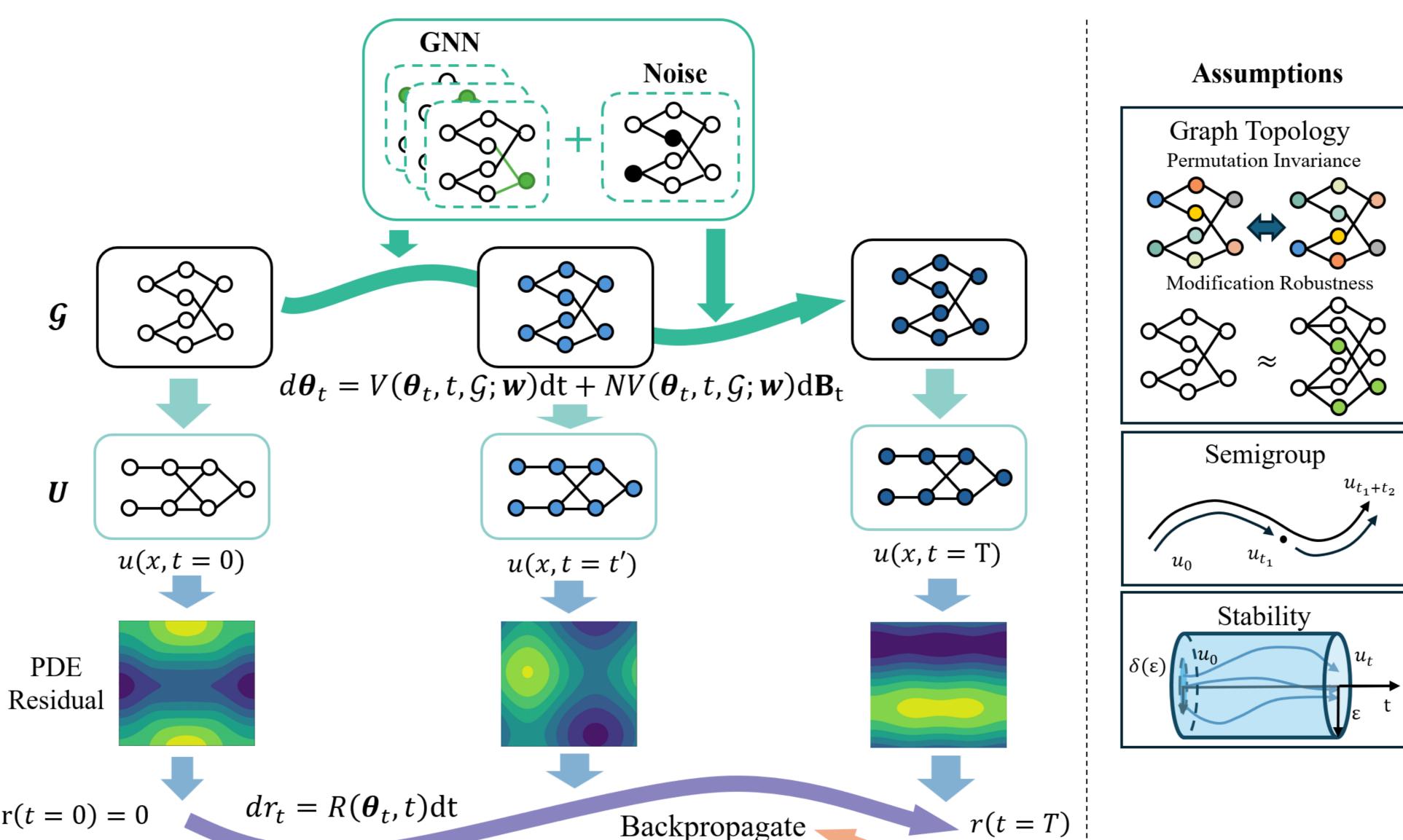
Introduction

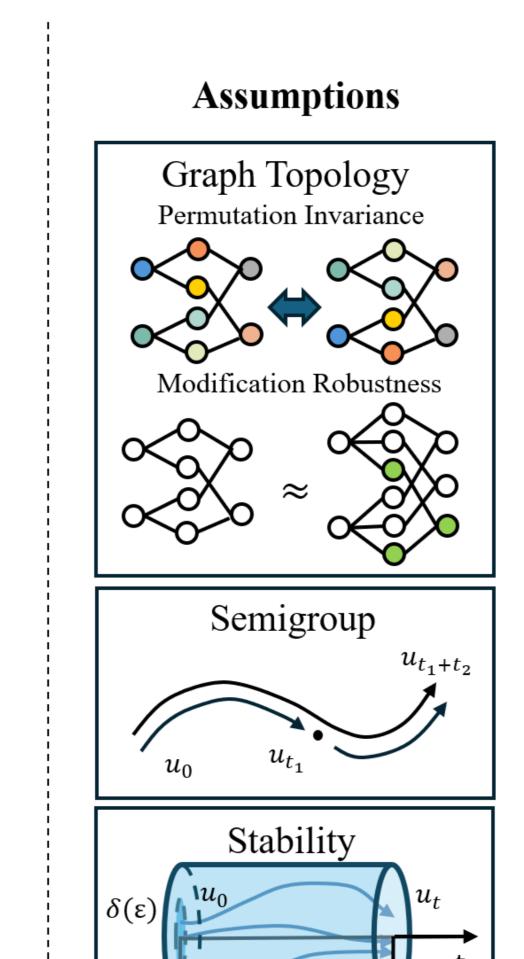
- Background: Curse of Dimension
- •Task: Operator learning for high-dim PDEs.
- Assumptions:
- > Graph topology: subnetwork robustness
- > Semigroup: continuous solution
- > Stability: perturbation insensitive
- •Model:
- Surrogate subnetwork
- GNN hypernetwork
- > Evolution in parameter space
- > Stochastic noise.

Method	Graph Topology	Semigroup	Stability	
NODE (Gaby & Ye, 2024)	No	Yes	No	
PINO (Li et al., 2024)	No	No	Yes	
SINGER (ours)	Yes	Yes	Yes	

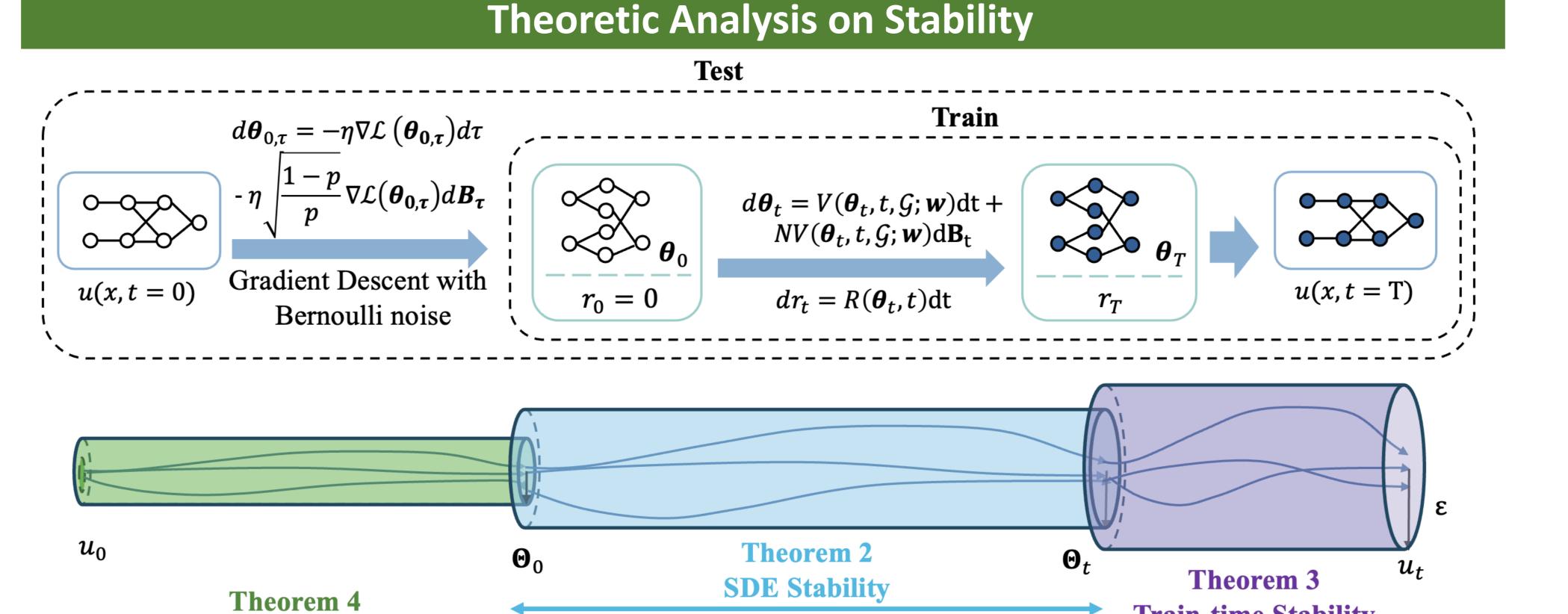
- •Datasets: 8 PDEs with 5,10,15,20 dimensions.
- Codes: github.com/FengMingquan-sjtu/SINGER

SINGER Architecture and Assumptions



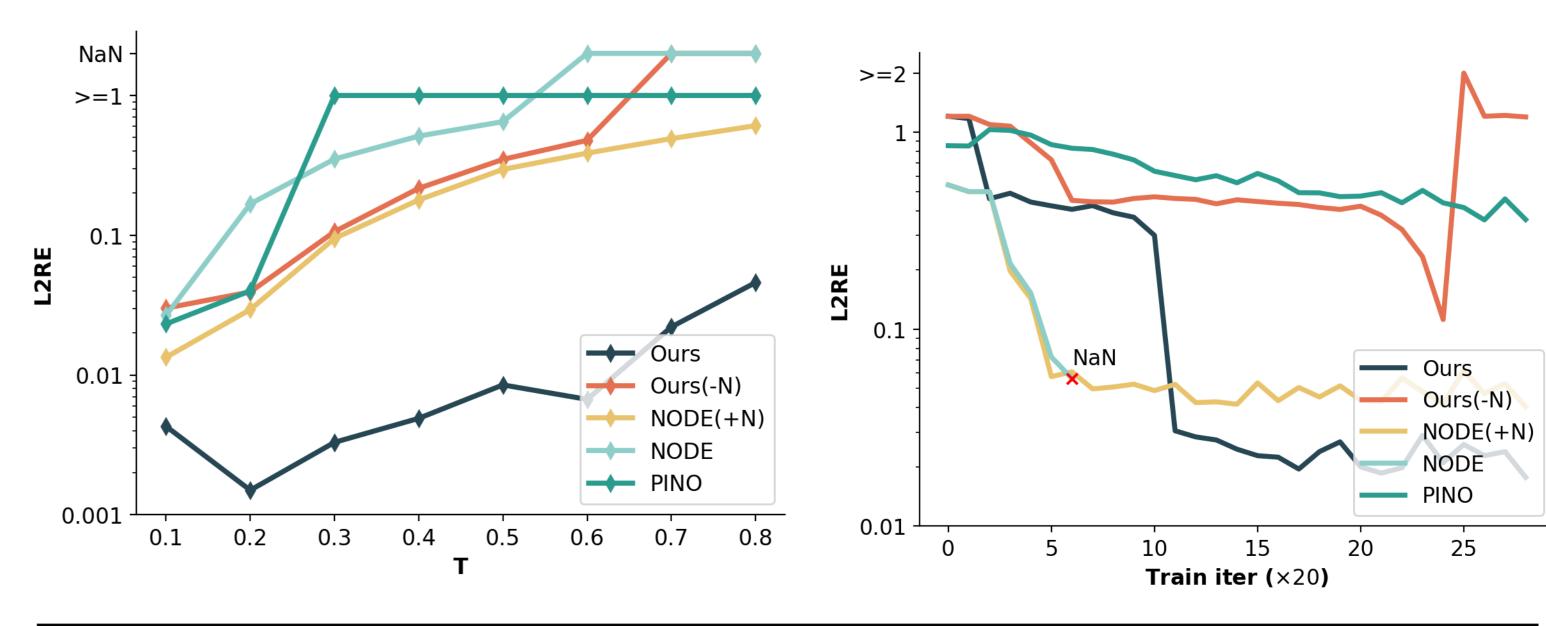


Train-time Stability



Test-time Stability

Numerical Results on High-dim PDEs



Eqn	Heat				HJB			
Dim	d=5	d=10	d=15	d=20	d=5	d=10	d=15	d=20
SINGER	0.0045	0.0041	0.0046	0.0175*	0.0036	0.0027	0.0099	0.0052
SINGER(-N)	0.0137*	0.0294*	0.0371*	0.0507	0.0037	0.0027	0.0106	0.0047
NODE	0.0194	0.0342*	0.0502*	0.0957*	0.0042	0.0090	0.0280	0.0602
NODE(+N)	0.0213	0.0220	0.0196	0.0751	0.0040	0.0041	0.0197	0.0398
PI-NO	0.0543	0.0229	0.0296	0.0567	0.1075	0.5387	0.9386	3.5078

