Filtered not Mixed: Filtering-Based Online Gating for Mixture of Large Language Models

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Introduction

- ▶ MoE models are state-of-the-art in LLMs for static tasks (Mixtral, Gemini, DBRX, ...)
- ▶ don't leverage the *temporal structure* of time-series data
- i.e., at every step we get feedback about the performance of each expert
- ▶ instantly feeding this back to the gating mechanism we derive an online adaptive MoE model, the Mixture-of-Experts Filter (MoE-F)

Problem formulation

- ▶ N pre-trained expert models $F = (f^{(1)}, ..., f^{(N)})$
- we assume that at any time t one of them is the true best expert specified by $w_t \in \{0,1\}^N$ with $|w_t|=1$
- w is the unobserved signal process, assumed to be a hidden Markov process

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- \triangleright w is the unobserved signal process, assumed to be a hidden Markov process
- ▶ we observe the target process Y (the time series to be predicted), assumed to be

$$Y_t = Y_0 + \underbrace{\int_0^t w_s^\top F(x_{[0,s]}) \, ds}_{\text{Best Expert Estimate}} + \underbrace{\int_0^t dW_s}_{\text{Idiosyncratic Residual Noise}}$$
 (1)

- problem of selecting the best expert is to filter w from observations of Y, F
- b this is a continuous-time finite state-space stochastic filtering problem (Wonham, 1964)
- we leverage the Wonham-Shiryaev filter, which is a closed-form recursive solution

MoE Filtering Algorithm

 \blacktriangleright 2 step approach applied at any time step t, using MSE or BCE loss function ℓ

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Step 1: Optimal Parallel Filtering

- ▶ for each expert $f^{(n)}$ compute its running performance $\ell_t^{(n)} = \ell(Y_t, f^{(n)}(x_{[0:t-1]}))$
- ightharpoonup and consider the filtering problem of optimally estimating w_t given its loss $\ell_t^{(n)}$
- ▶ solve these N stochastic filtering problems (in parallel) with the Wonham-Shiryaev filter
- lacksquare this yields the estimates $\pi_t^{(n)} = \left(\mathbb{P}\left(w_t = e_i \mid \mathcal{F}_t^{(n)}\right)\right)_{i=1}^N$, where $\mathcal{F}_t^{(n)} = \sigma\{\ell_s^{(n)}\}_{0 \leq s \leq t}$

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Step 2: Robust Aggregation

- lacktriangle each filter implies an individual (a posteriori) prediction $\hat{Y}_t^{(n)} = (\pi_t^{(n)})^{\top} F(x_{[0:t-1]})$
- ▶ loss scores $s_n = \ell \big(Y_t, \, \hat{Y}_t^{(n)} \big)$ used to define aggregation weights $\bar{\pi}_t^n = \frac{e^{-\lambda \, s_n}}{\sum_{i=1}^N e^{-\lambda \, s_i}}$
- **>** they aggregate experts into single (a posteriori) prediction $\hat{Y}_t = \sum_{n=1}^N \bar{\pi}_t^n \hat{Y}_t^{(n)}$
- ▶ aggregated estimate of signal w is $\hat{\pi}_t = \sum_{n=1}^N \bar{\pi}_t^n \pi_t^{(n)}$, used to mix experts at t+1



Theoretical Guarantees

Theorem 1 (informal)

The individual estimates $\pi_t^{(n)}$ of Step 1 are optimal filters (in L^2 sense) of the signal w.

Theorem 2 (informal)

The aggregation weights $\bar{\pi}_t$ optimally aggregate the loss scores under entropic regularization.

Dataset:

- test split of US equity market movement dataset (Saqur et al., 2024)
- ► features at t: market's current contextual information, i.e., relevant financial news headlines, market's financial numerics (OHLCV & technical indicators of past few days)
- ▶ labels at t: movement of \$SPY at t+1 in { 'Fall', 'Neutral', 'Rise' }

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Experts:

- ► Llama-2, Llama-3, Mixtral, DBRX-Intruct, GPT-4o
- \triangleright each expert LLM is prompted at each t to predict the label given the features

Results:

- ▶ all single LLM experts have similar performance (none is outstanding)
- ▶ MoE-F yields large performance increase of 17% real and 48.5% relative F1 measure improvement

| | LLM Experts | | | | | | | Experts Filter |
|--------------------|--------------------|---------------------|------------------------|-------------------------|-------------------------------|------------------|------------------|-----------------|
| $Metrics \uparrow$ | Llama-2 7b-chat | Llama-2 70b-chat | Llama-3 8B-Instruct | Llama-3 70B-Instruct | Mixtral-8x7B Instruct-v0.1 | DBRX Instruct | OpenAl GPT-4o | MoE-F (ours) |
| F1 | 0.22 | 0.33 | 0.35 | 0.20 | 0.34 | 0.34 | 0.34 | 0.52 |
| Acc | 0.27 | 0.37 | 0.39 | 0.30 | 0.33 | 0.34 | 0.37 | 0.57 |
| Precision | 0.35 | 0.33 | 0.31 | 0.32 | 0.36 | 0.36 | 0.33 | 0.61 |
| Recall | 0.27 | 0.37 | 0.39 | 0.30 | 0.33 | 0.34 | 0.37 | 0.57 |

References I

Raeid Saqur, Ken Kato, Nicholas Vinden, and Frank Rudzicz. Nifty financial news headlines dataset, 2024. Manuscript under review.

W Murray Wonham. Some applications of stochastic differential equations to optimal nonlinear filtering. *Journal of the Society for Industrial and Applied Mathematics, Series A: Control*, 2 (3):347–369, 1964.