Fengbo: a Clifford Neural Operator pipeline for 3D PDEs in Computational Fluid Dynamics

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Problem: 3D PDEs over irregular geometries

Navier-Stokes Equations

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \psi) = 0 \quad (1)$$

$$\rho \frac{\partial \psi}{\partial t} + \rho(\psi \cdot \nabla)\psi = -\nabla \phi + \mu \nabla^2 \psi + \mathbf{f} \quad (2)$$

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Problem Statement

Estimate pressure field $\phi(\mathbf{x}) : \Omega_D \subset \mathbb{R}^3 \rightarrow \mathbb{R}$ and velocity field $\psi(\mathbf{x}) : \Omega_D \subset \mathbb{R}^3 \rightarrow \mathbb{R}^3$

Geometric Algebra Networks

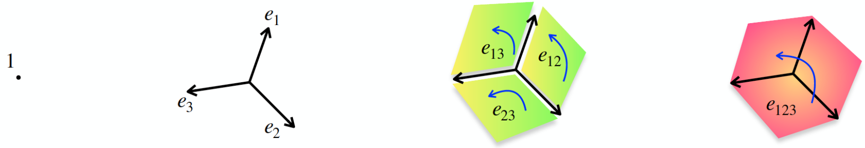
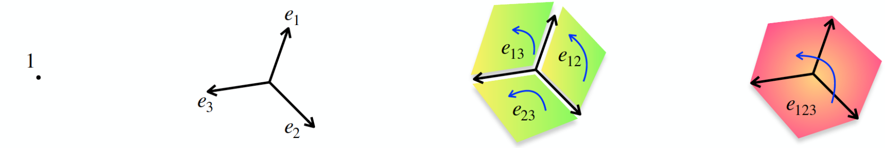


Figure 1: Elements of $G(3,0,0)$, the 3D Euclidean space.

Why Hypercomplex Neural Networks in Geometric Algebra (GA)?

- **Compact**
- **Interpretable**
- **Lower Complexity**
- **Better Generalisability**



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We embed **inputs (i.e. geometry)** and **outputs (i.e. physical quantities)** as objects in G(3,0,0), known as *multivectors*.

Strategy: multivectors defined over regular grids

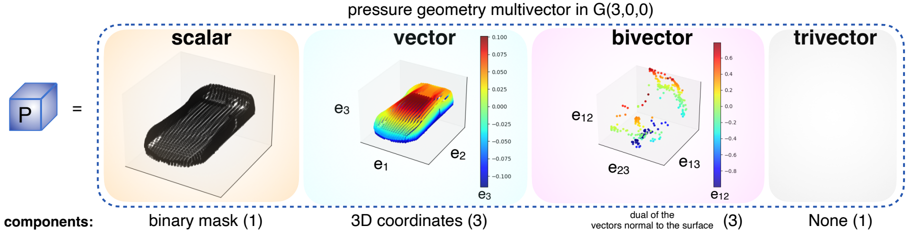

Figure 2: Input multivector P

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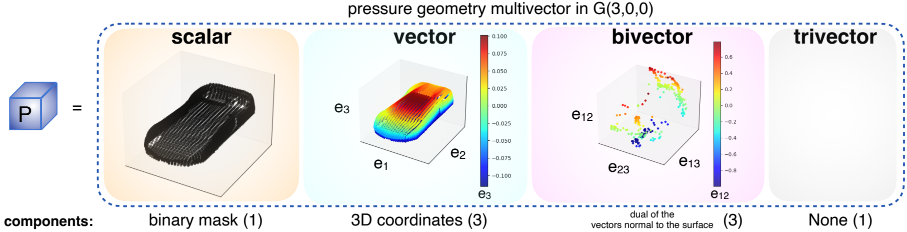

Figure 2: Input multivector P

Input: (geometry) multivector P

$$P = m_p + \mathbf{p} + B = \underbrace{m_p}_{\text{scalar}} + \underbrace{p_1 e_1 + p_2 e_2 + p_3 e_3}_{\text{vector}} + \underbrace{B_{12} e_{12} + B_{13} e_{13} + B_{23} e_{23}}_{\text{bivector}} \quad (3)$$

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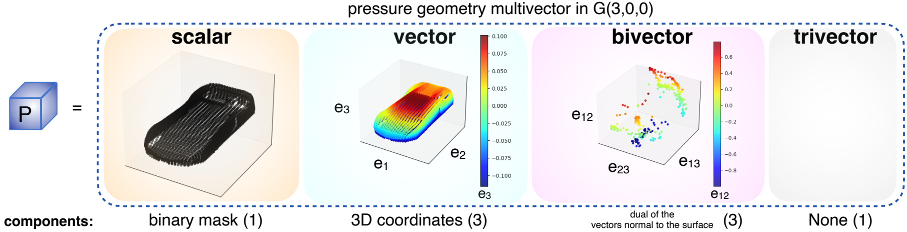

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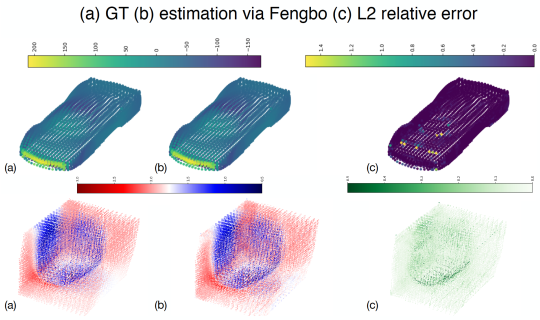

Figure 3: Estimated pressure and velocity fields ϕ, ψ for the *ShapeNet Car* dataset.

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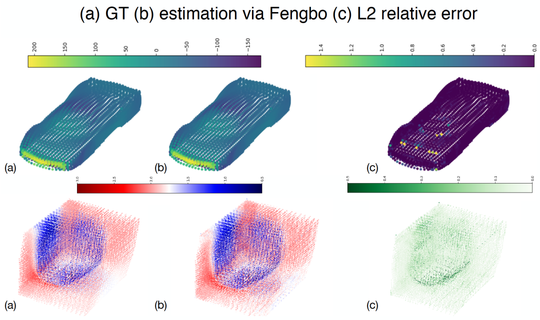

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Output: (physics) multivector P'

$$P' = \phi + \psi = \underbrace{\phi}_{\text{scalar}} + \underbrace{\psi_1 e_1 + \psi_2 e_2 + \psi_3 e_3}_{\text{vector}} \tag{4}$$

The Fengbo pipeline

Module	Input	Output	Purpose
1. 3D Clifford Geometry blocks	$\mathbf{P} = \{P_i\}_{i=1}^{N_g}$	$\mathbf{Q} = \{Q_i\}_{i=1}^{N_g}$	local, upsample, grade mixing
2. 3D Clifford FNO	$Q = \sum_{i=1}^{N_g} Q_i$	Q'	global, PDE modelling
3. 3D Clifford Physics blocks	Q'	$\mathbf{P}' = \{P'_i\}_{i=1}^{N_p}$	local, downsample, grade mixing

Figure 4: Fengbo $P' = \Xi(P)$

- Reduced parameters and computational complexity.

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- Multi-quantity estimation possible.
- End-to-end interpretability.