

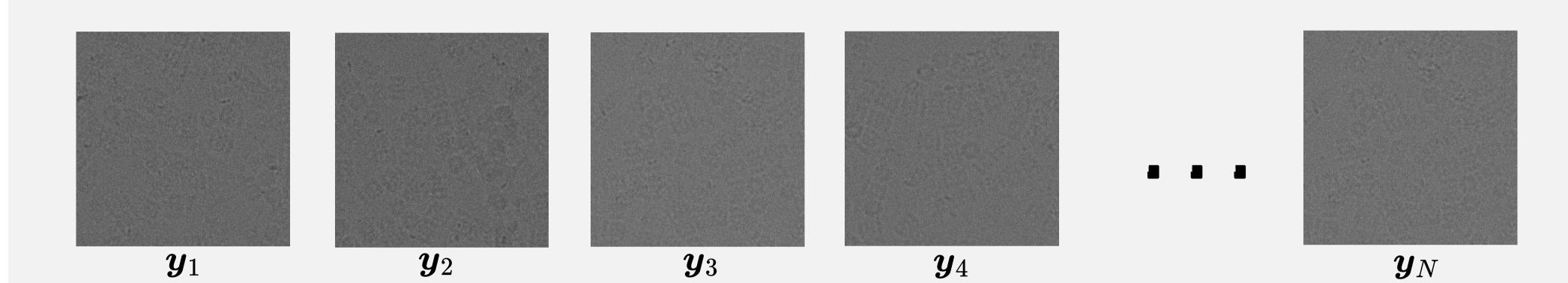
UNSURE: self-supervised learning with Unknown Noise level Stein's Unbiased Risk Estimate



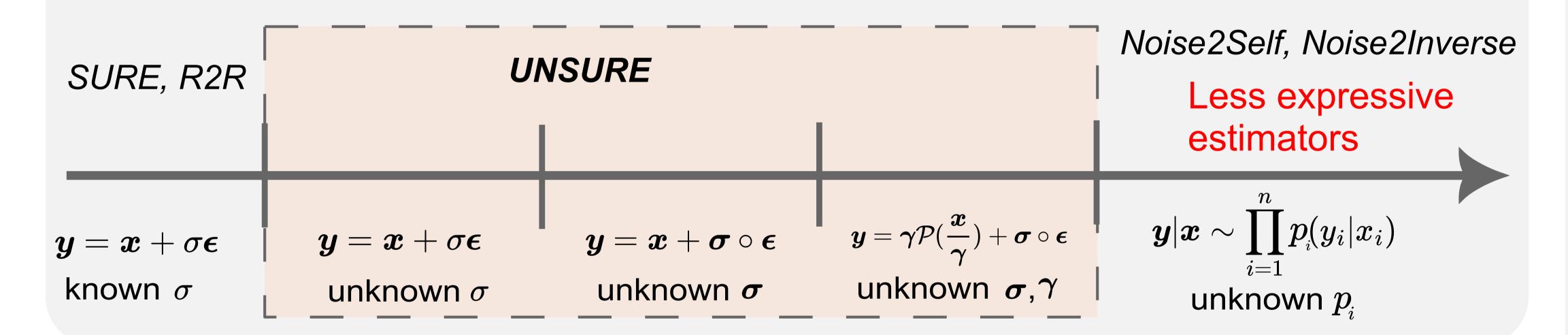


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Q. What can we learn from noisy images alone?



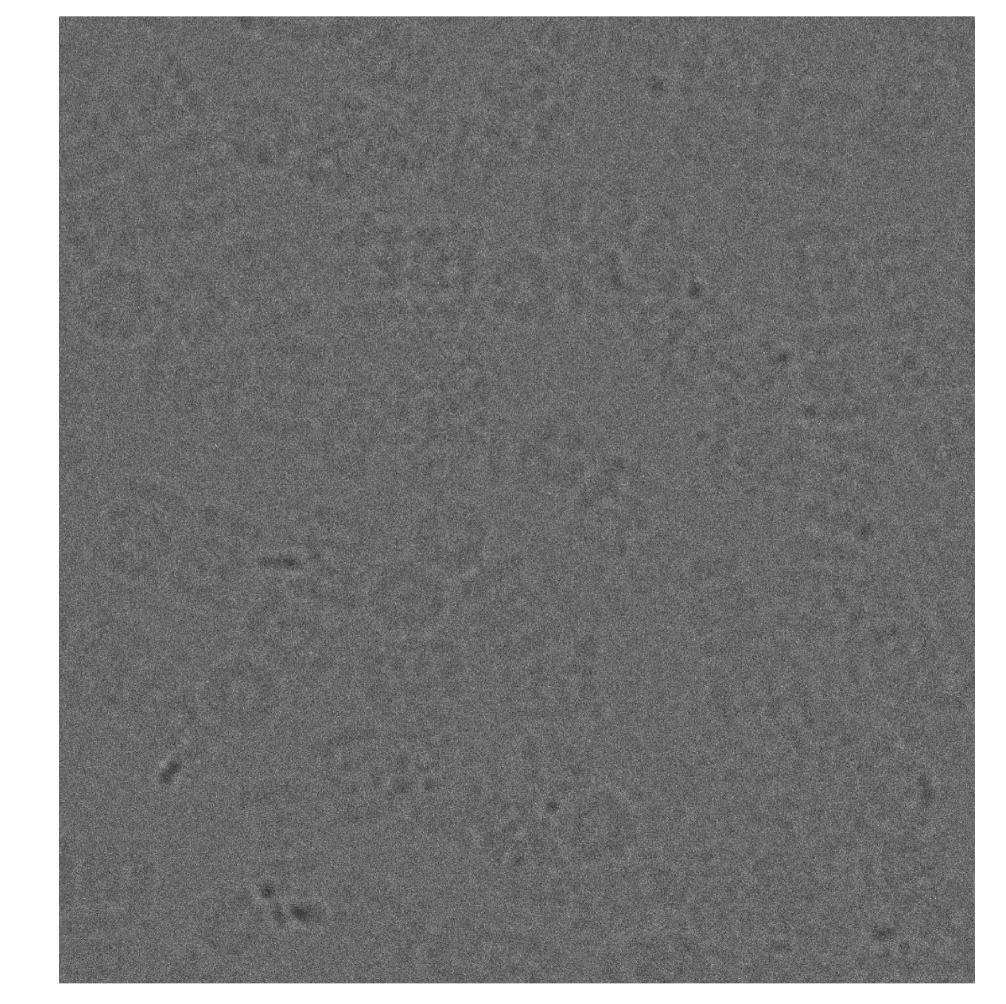
A. Depends on knowledge about the noise distribution



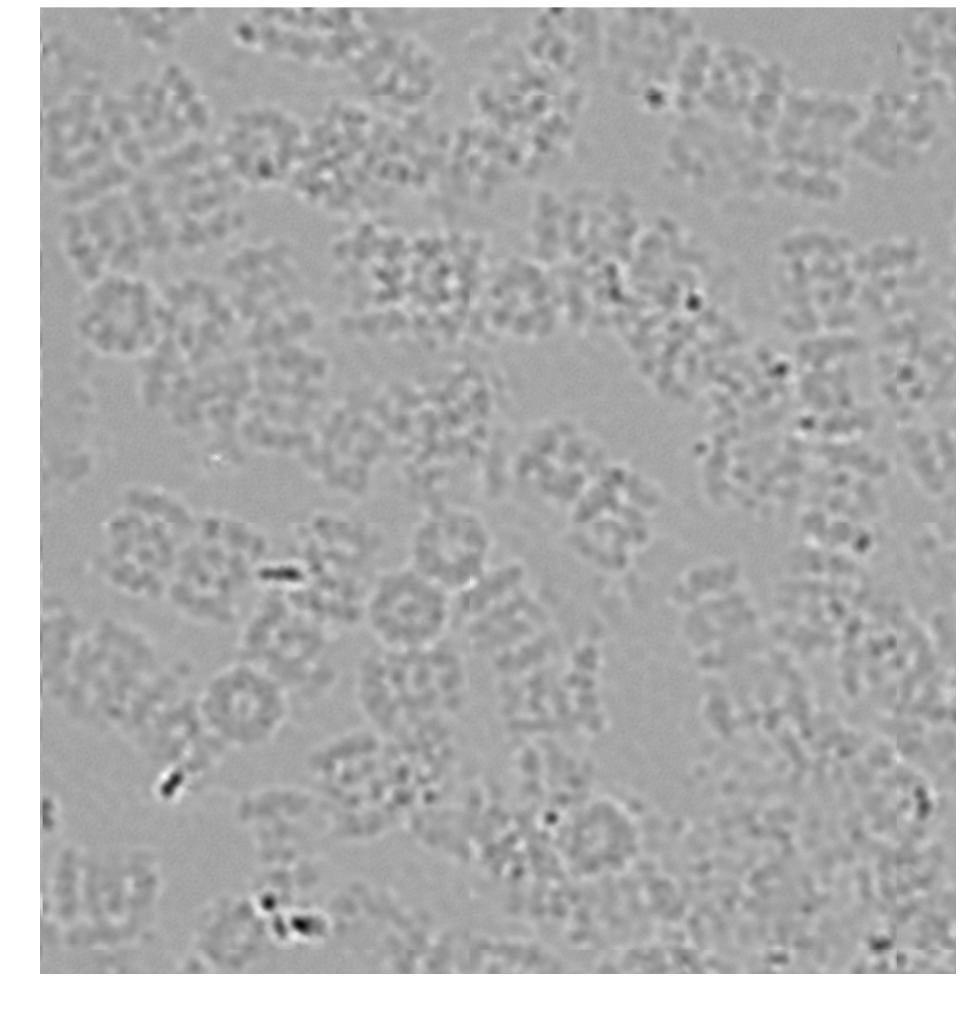


PSNR [dB] in yellow

Real Cryo-EM data



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Q. What loss should I minimize?

A. Consistency with constraints

$$\min_{f} \mathbb{E}_{oldsymbol{y}} \|f(oldsymbol{y}) - oldsymbol{y}\|^2 ext{ s.t. constraint } f = 0$$

a) Gaussian noise unknown σ

$$\sum_{i=1}^n \mathbb{E}_{m{y}} rac{\partial f_i}{\partial y_i}(m{y}) = 0$$

b) Poisson Gaussian unknown (σ, γ)

$$\sum_{i=1}^n \mathbb{E}_{m{y}} rac{\partial f_i}{\partial y_i}(m{y}) = 0 \quad \sum_{i=1}^n y_i rac{\partial f_i}{\partial y_i}(m{y}) = 0$$

c) Gaussian noise unknown Σ

Possible covariances:
$$\{\Sigma=\sum_{j=1}^J \eta_j\Psi_j,\,m{\eta}\in\mathbb{R}^J\}$$
 $\mathbb{E}_{m{y}}\operatorname{tr}(\Psi_j
abla f(m{y}))=0\ \ ext{for}\ j=1,\ldots,J$

d) Other models ... (see paper)

Q. Why these losses?

A. Because of SURE

$$\min_{f} \; \mathbb{E}_{oldsymbol{y}} \|f(oldsymbol{y}) - oldsymbol{y}\|^2 \; ext{ s.t. constraint } f = 0 \qquad \min_{f} \; \mathbb{E}_{oldsymbol{y}} \|f(oldsymbol{y}) - oldsymbol{y}\|^2 + 2\sigma^2 \sum_{i=1}^n \mathbb{E}_{oldsymbol{y}} rac{\partial f}{\partial y_i}(oldsymbol{y})$$

Q. What's the performance?

A. Almost as supervised

a) Generalization of Tweedie

$$f(oldsymbol{y}) = oldsymbol{y} + rac{n
abla \log p(oldsymbol{y})}{\mathbb{E}_{oldsymbol{y}} \|
abla \log p(oldsymbol{y}) \|^2}$$

b) Expected performance

$$\mathbb{E}_{oldsymbol{x},oldsymbol{y}} rac{1}{n} \|f(oldsymbol{y}) - oldsymbol{x}\|^2 = ext{MMSE} + \sigma^2 \sum_{j=2}^{\infty} (rac{ ext{MMSE}}{\sigma^2})^j.$$

c) Upper bound noise level

$$rac{n}{\mathbb{E}_{oldsymbol{y}} \|
abla \log p(oldsymbol{y})\|^2} = rac{\sigma^2}{1 - rac{ ext{MMSE}}{\sigma^2}} \geq \sigma^2$$

Q. Implementation?

A. Lagrange multipliers

$$\min_{f} \max_{m{\eta}} \mathbb{E}_{m{y}} \|f(m{y}) - m{y}\|^2 + 2 \operatorname{tr} \Bigl(\Sigma_{m{\eta}} rac{\partial f}{\partial m{y}}(m{y}) \Bigr)$$

Divergence approximation

$$oldsymbol{b} \sim \mathcal{N}(oldsymbol{0}, I) \qquad 0 < au \ll 1$$

$$ext{tr} \Big(\Sigma_{m{\eta}} rac{\partial f}{\partial m{y}}(m{y}) \Big) pprox rac{(\Sigma_{m{\eta}} m{b})^ op}{ au} \Big(f(m{y} + au m{b}) - f(m{y}) \Big)$$

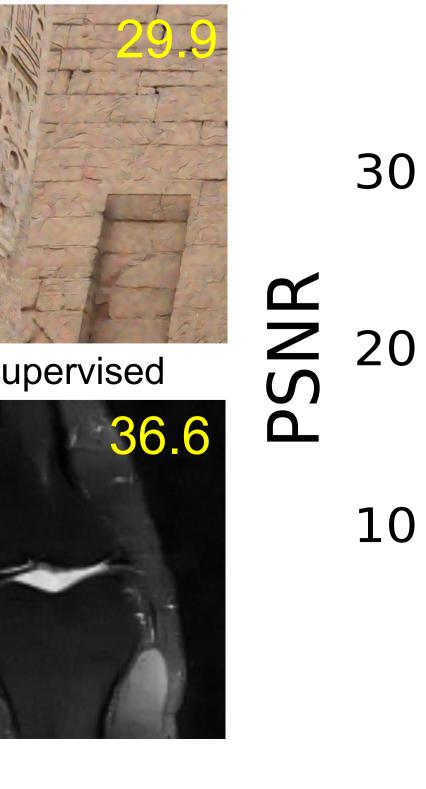
Algorithm 1 UNSURE loss.

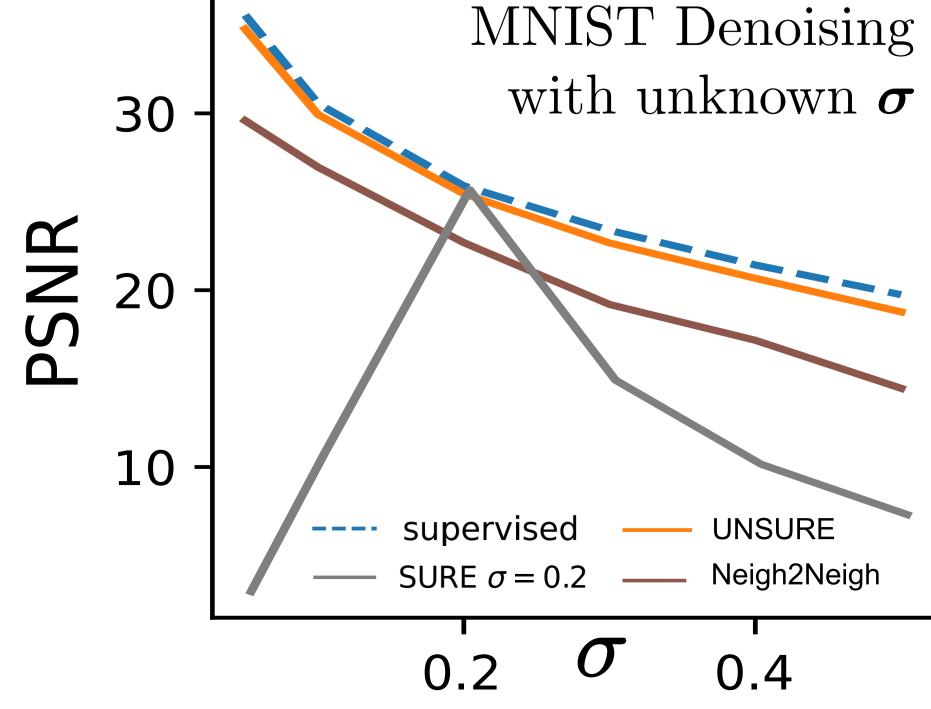
Require: step size α , momentum μ residual $\leftarrow \|f_{\boldsymbol{\theta}}(\boldsymbol{y}) - \boldsymbol{y}\|^2$ $oldsymbol{b} \sim \mathcal{N}(oldsymbol{0}, oldsymbol{I})$

$$\operatorname{div} \leftarrow \frac{(\boldsymbol{\Sigma}_{\boldsymbol{\eta}} \boldsymbol{b})^{\top}}{\tau} \left(f_{\boldsymbol{\theta}} (\boldsymbol{y} + \tau \boldsymbol{b}) - f_{\boldsymbol{\theta}} (\boldsymbol{y}) \right) \\ \operatorname{loss} \leftarrow \operatorname{residual} + \operatorname{div}$$

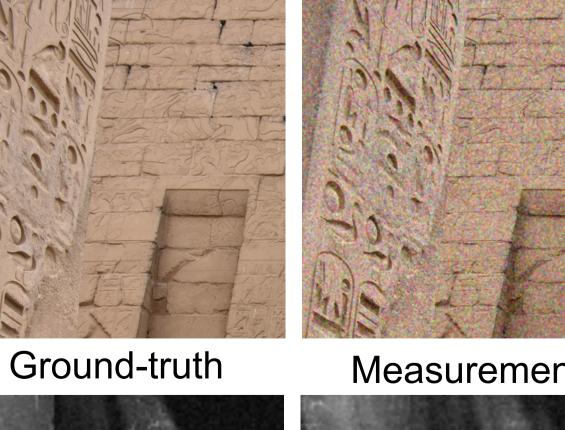
$$m{g} \leftarrow \mu m{g} + (1 - \mu) \, rac{\partial \mathrm{div}}{\partial m{\eta}}$$
 Added lines vs SURE return $\mathrm{loss}(m{ heta})$

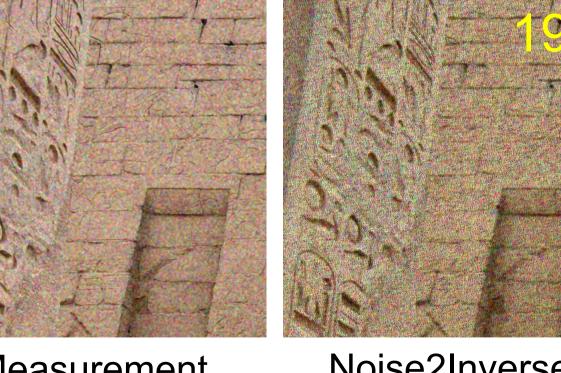
vs SURE

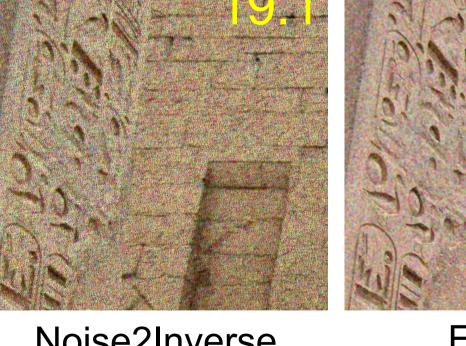


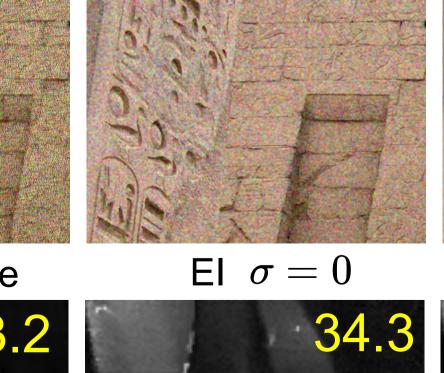


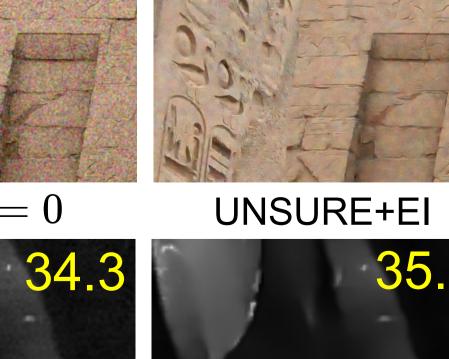


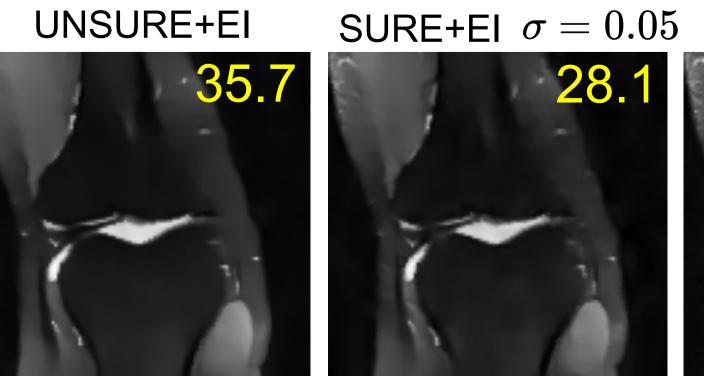












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