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# Unlocking Point Processes through Point Set Diffusion

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## TL;DR

- A diffusion-based latent variable model for sets on general metric spaces
- We learn stochastic interpolation paths between data and random point sets
- This parametrization enables both efficient, parallel sampling and flexible generation for complex conditional tasks

## Motivation

Modelling the complex interactions between points commonly leverages the conditional intensity function:

$$\lambda(x) = \lim_{\delta \to 0} \frac{P\{N(B_\delta(x)) = 1 \mid C(N(\mathbb{R} \setminus B_\delta(x)))\}}{|B_\delta(x)|}$$

where $C$ represents the information of all other points outside the ball $B_\delta$ centered at x.

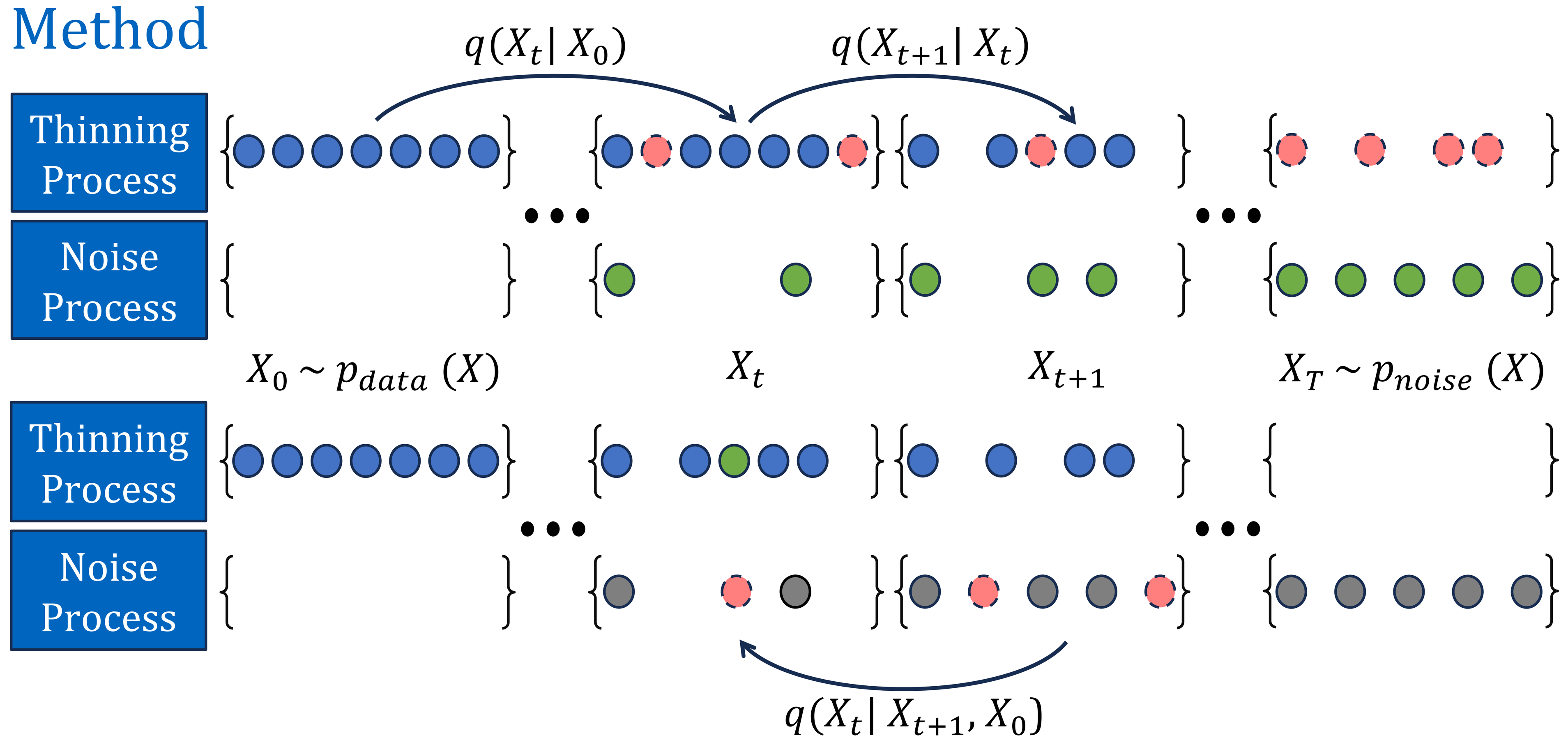
Effectively **modelling** and **sampling** point processes with the conditional intensity is a fundamental problem having led to:

- **Oversimplified parametrizations**
- **Approximations**
- **Restrictive factorizations**
- **Limited evaluations**

**How to overcome the conditional intensity function?**

We propose to model point processes by directly learning to stochastically interpolate between data and noise point sets

## Method




### Approximating the posterior

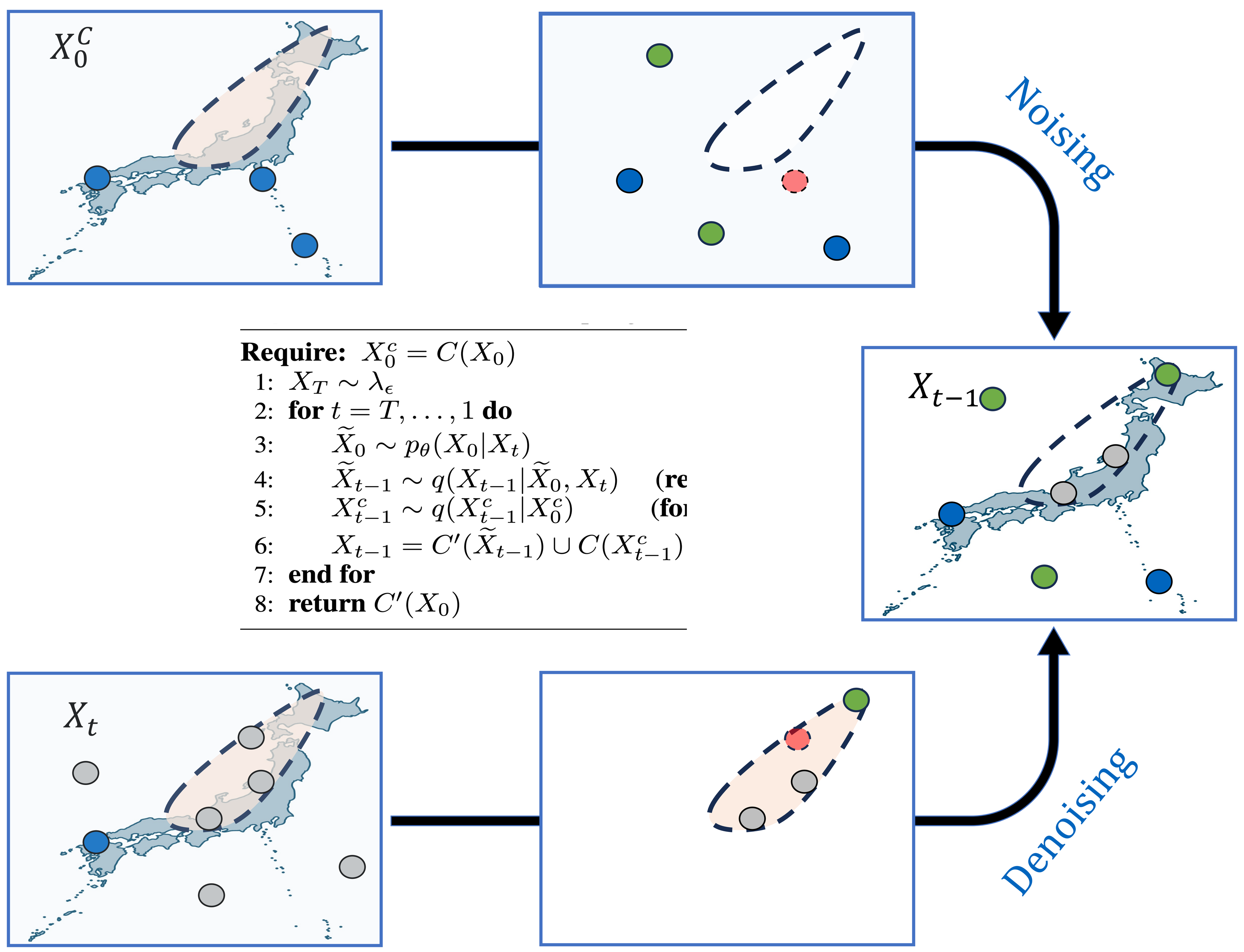
$$q_\theta(X_t \mid X_{t+1}, X_0) = \int q(X_t \mid X_{t+1}, \tilde{X}_0)\, p_\theta(\tilde{X}_0 \mid X_{t+1})$$

**Model** $X^0 \mid X_{t+1}$

Classify: $X_0 \cap X_{t+1}$

Prob. Predict: $X_0 \setminus X_{t+1}$

### Conditional sampling




**Require:** $X_0^c = C(X_0)$
$X_T \sim \lambda_\epsilon$
**for** $t = T, \ldots, 1$ **do**
$\widetilde{X}_0 \sim p_\theta(X_0 \mid X_t)$
$\widetilde{X}_{t-1} \sim q(X_{t-1} \mid \widetilde{X}_0, X_t)$ **(re**
$X_{t-1}^c \sim q(X_{t-1}^c \mid X_0^c)$ **(fo**
$X_{t-1} = C'(\widetilde{X}_{t-1}) \cup C(X_{t-1}^c)$
**end for**
**return** $C'(X_0)$

## Results

### SPP

Table 1: Density estimation results on the hold-out test set for SPPs, averaged over three random seeds (**bold** best and <u>underline</u> second best).

| | Earthquakes | | Covid NJ | | Citybike | | Pinwheel | |
|---|---|---|---|---|---|---|---|---|
| | SL(↓) | MMD(↓) | SL(↓) | MMD(↓) | SL(↓) | MMD(↓) | SL(↓) | MMD(↓) |
| Log-Gaussian Cox | <u>0.047</u> | <u>0.214</u> | <u>0.209</u> | <u>0.340</u> | 0.104 | <u>0.336</u> | **0.017** | <u>0.285</u> |
| Regularized Method | 2.361 | 0.391 | 0.255 | 0.411 | <u>0.097</u> | 0.342 | <u>0.039</u> | 0.411 |
| Point Set Diffusion | **0.038** | **0.173** | **0.199** | **0.268** | **0.056** | **0.092** | **0.017** | **0.099** |

Table 2: Conditional generation results on the hold-out test set for SPP, averaged over three random seeds (**bold** best).

| | Earthquakes | | Covid NJ | | Citybike | | Pinwheel | |
|---|---|---|---|---|---|---|---|---|
| | MAE(↓) | WD(↓) | MAE(↓) | WD(↓) | MAE(↓) | WD(↓) | MAE(↓) | WD(↓) |
| Regularized Method | 30.419 | 0.162 | 16.075 | 0.148 | 7.740 | 0.115 | 3.547 | 0.150 |
| Point Set Diffusion | **4.651** | **0.106** | **5.056** | **0.119** | **3.498** | **0.085** | **2.256** | **0.122** |

### STPP

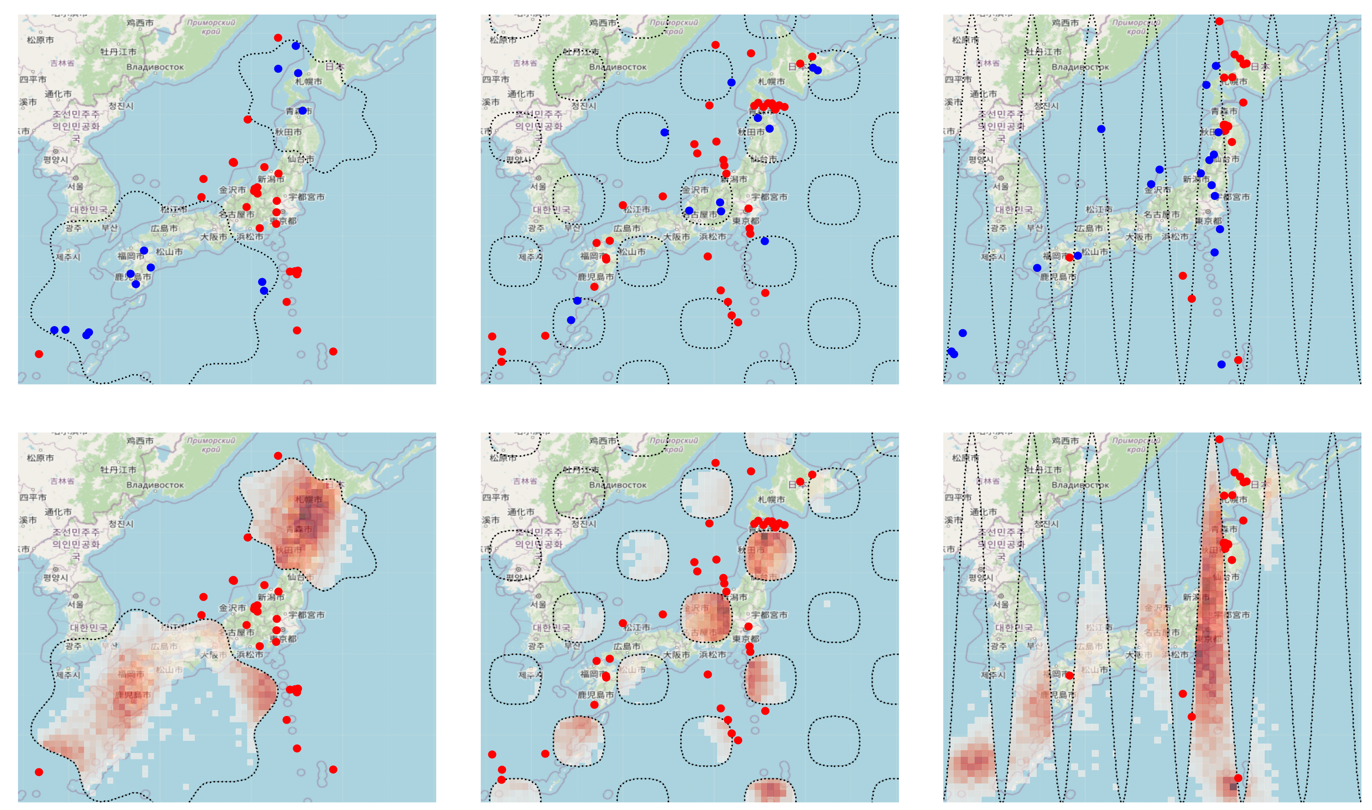
Table 3: Density estimation results on the hold-out test set for STPP, averaged over three random seeds (**bold** best and <u>underline</u> second best).

| | Earthquakes | | Covid NJ | | Citybike | | Pinwheel | |
|---|---|---|---|---|---|---|---|---|
| | SL(↓) | MMD(↓) | SL(↓) | MMD(↓) | SL(↓) | MMD(↓) | SL(↓) | MMD(↓) |
| DeepSTPP | 0.105 | 0.266 | **0.169** | 0.166 | 3.257 | 0.677 | 1.067 | 0.197 |
| DiffSTPP | 0.088 | 0.064 | 0.332 | <u>0.146</u> | <u>0.560</u> | 0.611 | 0.196 | <u>0.055</u> |
| AutoSTPP | <u>0.073</u> | <u>0.062</u> | 0.364 | 0.280 | 0.598 | <u>0.331</u> | <u>0.127</u> | 0.147 |
| Point Set Diffusion | **0.042** | **0.023** | <u>0.189</u> | **0.043** | **0.032** | **0.020** | **0.023** | **0.020** |

Table 4: Forecasting results on the hold-out test set for STPP, averaged over three random seeds (**bold** best and <u>underline</u> second best).

| | Earthquakes | | Covid NJ | | Citybike | | Pinwheel | |
|---|---|---|---|---|---|---|---|---|
| | MAE(↓) | CD(↓) | MAE(↓) | CD(↓) | MAE(↓) | CD(↓) | MAE(↓) | CD(↓) |
| DeepSTPP | <u>10.154</u> | <u>11.211</u> | **6.264** | **8.492** | 127.968 | 125.747 | 18.651 | 15.792 |
| DiffSTPP | 16.027 | 17.466 | 18.822 | 14.302 | <u>7.516</u> | <u>8.460</u> | <u>14.461</u> | <u>13.062</u> |
| Point Set Diffusion | **7.407** | **10.458** | <u>7.293</u> | <u>10.865</u> | **5.928** | **7.225** | **6.341** | **6.437** |

### Complex conditional sampling examples



**Paper and code are available here:**