DEEP CLUSTERING USING ADVERSARIAL NET BASED CLUSTERING LOSS

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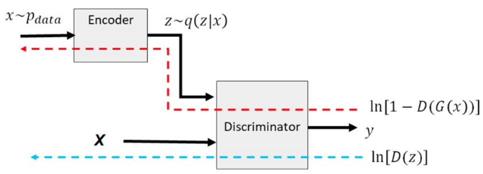
1.1 Literature - Different strategies of using Adversarial net

Method	Adversarial net	Task X
GAN	Is x from the ground truth or randomly generated? $\begin{cases} \text{if } x \sim p(\text{data}),\ T=1 \\ \text{elseif } x \sim p(g),\ T=0 \end{cases}$	$x \sim p(data)$
AAE	Is z from a Gaussian prior? $\begin{cases} \text{if } z \sim \mu + \sigma \cdot \mathcal{N}(0,1),\ T=1 \\ \text{elseif } z \sim q(z \mid x),\ T=0 \end{cases}$	$z \sim p(z)$
DASC	Does z fall along the assigned subspace? $\begin{cases} \text{if } z \sim p(\psi^*),\ T=1 \\ \text{elseif } z \sim p(\hat{\psi}),\ T=0 \end{cases}$	$z \sim p(\psi^*)$
DCAN	Does z belong to the assigned cluster? $\begin{cases} \text{if } z \sim p(z \mid \theta^*),\ T=1 \\ \text{elseif } z \sim q(z \mid x),\ T=0 \end{cases}$	$z \sim p(z \mid \theta^*)$

1.2 Adversarial net based "X"

Their encoder-discriminator path is common, but discriminator input X is different.

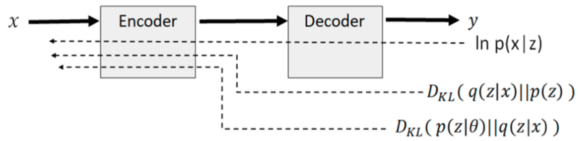


1.3 Proposed DCAN - Adversarial net based "Deep Clustering"

$$L_{DCAN} = \left\{ \frac{1}{N}\sum_{n=1}^{N} \underset{x^{(n)} \sim p_{data}}{E} \left[\ln\left(1 - D\left(G(x^{(n)})\right)\right) \right] \right\}_{T=0} + \left\{ \frac{1}{N}\sum_{n=1}^{N} \underset{z^{(n)} \sim p(z|\theta^*)}{E} \left[\ln D\left(z^{(n)}\right) \right] \right\}_{T=1}$$

2. Relating Deep Clustering to Adversarial Net using Lemmas1 and Lemma2

2.1 Lemma1 – Deep clustering as JSD loss



VAE based deep clustering has the loss functions at encoder and decoder as shown

$$\mathcal{L}^{VAED} = \mathcal{L}^{VAE} - \lambda_3 \cdot D_{KL}\left(p(z_n \mid \theta) \parallel q(z_n \mid x_n)\right)$$

Deep clustering loss refers to the KLD between clustering and encoder

$$D_{KL}\left(\, p(z \mid \theta) \parallel q(z \mid x) \,\right)$$

Alternative, we can represent deep clustering loss as the JSD between two KLDs

$$D_{JS}\left(\, p(z \mid \theta) \parallel q(z \mid x) \,\right) = \frac{1}{2} D_{KL}\left(p \parallel \frac{p+q}{2}\right) + \frac{1}{2} D_{KL}\left(q \parallel \frac{p+q}{2}\right)$$

2.2 Lemma2 – Adversarial net as JSD based deep clustering loss

However, no closed-form for JSD due to mixture distribution $\frac{p+q}{2}$

Instead, we can bypass JSD closed form, using adversarial net

$$\begin{aligned} & D_{JS}\left(p(z \mid \theta^*) \parallel q(z \mid x)\right) \\ = & \underset{z \sim p(z|\theta^*)}{E}[\ln D(z)] + \underset{x \sim p_{data}}{E}[\ln(1 - D\{G(x)\})] \\ = & \underset{z \sim p(z|\theta^*)}{E}[\ln D(z)] + \underset{z \sim q(z|x)}{E}[\ln(1 - D\{z\})] \end{aligned}$$

Adversarial net approaches JSD at optimum, as below

$$\begin{aligned} \boldsymbol{LHS} : & E_{z\sim p}[\ln D(z)] + E_{x \sim p_{data}}[\ln(1 - D\{G(x)\})] \quad s.t.\ \{D = \tfrac{p}{p+q}\} \\ & = E_{z\sim p}[\log D(z)] + E_{z\sim q}[\log(1 - D(z))] \quad s.t.\ \{D = \tfrac{p}{p+q}\} \\ & = E_{z\sim p}\left[\ln \tfrac{p}{p+q}\right] + E_{z\sim q}\left[\ln \tfrac{q}{p+q}\right] \\ & = \int_z p \ln \tfrac{p}{p+q} + q \ln \tfrac{q}{p+q}\, dz \end{aligned}$$

$$\begin{aligned} \boldsymbol{RHS} : & D_{JS}(p \parallel q) = \tfrac{1}{2}\int p \ln \tfrac{2 \cdot p}{p+q} + q \ln \tfrac{2 \cdot p}{p+q}\, dz \\ & = \tfrac{1}{2}\int_z p\left\{\ln \tfrac{p}{p+q} + \ln 2\right\} + q\left\{\ln \tfrac{q}{p+q} + \ln 2\right\} dz \end{aligned}$$

$$LHS \leq 2RHS - 2\log 2.$$

3. Training DCAN

Discriminator weight update

$$\triangle w_\phi = \frac{\delta}{\delta w_\phi}\left\{ \frac{1}{N}\sum_{n=1}^{N} \underset{x^{(n)} \sim p_{data}}{E} \left[\ln\left(1 - D\left(G(x^{(n)})\right)\right) \right] \right\}$$

$$\triangle w_\phi = \frac{\delta}{\delta w_\phi}\left\{ \frac{1}{N}\sum_{n=1}^{N} \underset{z^{(n)} \sim p(z|\theta^*)}{E} \left[\ln D\left(z^{(n)}\right) \right] \right\}$$

Encoder weight update

$$\triangle w_\rho = \frac{\delta}{\delta w_\rho}\left\{ \frac{1}{N}\sum_{n=1}^{N} \underset{x^{(n)} \sim p_{data}}{E} \left[\ln\left(1 - D\left(G(x^{(n)})\right)\right) \right] \right\}$$

Cross entropy loss

$$D = y^T (1 - y)^{1-T}$$

4. Experimental Results

Table 2: ACC Benchmark

Approach	MNIST	Reuters10k	CIFAR10
ABC Song et al. (2013)	0.760	0.7019	0.435
DEC Xie et al. (2016)	0.843	0.7217	0.469
DC-GMM Tian et al. (2017)	0.8555	0.6906	-
AAE Makhzani et al. (2016)	0.8348	0.6982	-
IMSAT-RPT Hu et al. (2017)	0.896	0.719	0.455
KINGDRA Gupta et al. (2020)	**0.985**	0.705	0.546
DCAN (proposed)	0.8565	**0.7867**	**0.5844**